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Introduction to the Early Development of Mathematics (Book Review)

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Abstract
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An Introduction to the Early Development of Mathematics

Michael K. J. Goodman

Reviewed by Calvin Jongsma, on 03/28/2016

According to the book's back cover, "*An Introduction to the Early Development of Mathematics* is an ideal textbook for undergraduate courses on the history of mathematics and a supplement for elementary and secondary education majors." Whatever else it might be, it is not that. What it may be instead is a college textbook for a liberal arts mathematics course that focuses on elementary and recreational problem solving, using historical material as an organizing scheme and as exercise prompts. But even for that venue, I hesitate to recommend it: the history ought to be more accurate and better grounded in recent scholarship than this book's is, especially if it is to be used with mathematics education students. My criticism is not that it fails to be a comprehensive history of mathematics text, nor that it lacks mathematical depth. The author, Michael Goodman, readily admits to being "an armchair historian", not a researcher; but one can study the current secondary literature and some primary sources in translation and end up with a more faithful portrayal of the historical developments covered here.

Let me document my reservations. Because the low-level mathematics course that gave rise to this text is "designed primarily for non-technical students", the author chose to keep the mathematics simple and to restrict the history to developments before 1000 AD. I admire the book's attempt to present elementary mathematics in historical perspective — I think, for a number of reasons, that prospective middle school and secondary school mathematics teachers ought to know the historical background of what they will teach. But cutting the history off at year 1000 makes no historical sense to me, even if the focus is just on elementary mathematics. Stopping at 1637 would have been a far better choice: by that point Hindu-Arabic numeration and arithmetic was well established in Europe; decimal fractions had been developed, even if decimal measurement systems still languished; logarithms were introduced as a significant aid to astronomical calculations; mathematicians had determined methods for solving all cubics and quartics, launching complex number arithmetic in the process; elementary algebra was well on its way to becoming the symbolic field we now recognize it to be; and coordinate geometry had just made its double entrance in France. Stopping at 1000 means all of these important developments in elementary and secondary mathematics are missed.

Be that as it may, let's look at the book's treatment of various cultures' contributions to the development of elementary mathematics up to the self-imposed limit. The author discusses ancient Egyptian, ancient Chinese, Babylonian, classical Greek, early Hindu, and early Arabian mathematics, giving a chapter to each. In addition, he presents even earlier developments in a four-page chapter on "mathematical anthropology", and a later chapter is devoted to a discussion of how historians figured out what ancient mathematics was all about ("mathematical archeology").

In presenting the historical material from each culture, Goodman freely uses present-day algebraic notation and procedures to formulate problems and produce solutions via symbolic manipulation of equations. Given that this supposes a form of algebra that wasn't available for 600-plus years after the book's chosen terminal date, this approach doesn't help the reader understand the historical nature of the material being presented. Furthermore, it's not just that an anachronistic reformulation of the historical material is being used; the reader is left to wonder whether the methods being used can be located in some form in the culture being studied or whether they are only based in today's high school algebra and coordinate geometry. Far too often, it is the latter — which may be fine for a liberal arts problem-solving course, but not for a course in history of mathematics.

Related to this issue is the one of problem choice itself. Goodman offers quite a few problems (easy) and exercises (more difficult) for the reader to follow and work (giving hints for selected problems), but he invariably fails to identify where these come from. At times they may come from the culture's source material (but where?); more often they're newly minted to illustrate what sorts of mathematics were being considered back then and to give students additional practice in applying the solution method just illustrated. If it truly was the author's intent to whet his students' appetite for more in-depth presentations of the history of mathematics, though, as he says in the introduction, wouldn't specific references have been useful?

Goodman's choice of which cultures to consider was made because the mathematics done by them was not too difficult for his students to work with and because he could present it as a sort of puzzle for them to solve; but presumably they were also chosen because of the importance of those cultures themselves and because of the significance of their mathematics. One would expect, then, that Goodman would recognize the significance of the various Near-Eastern and Eastern cultures' contributions to our own Western understanding of mathematics. But the book's perspective remains too much mired in an older Eurocentric understanding of the history of mathematics. Goodman sees a Greek origin for almost everything of significance in pre-college mathematics: "Virtually, everything you learned in school about basic arithmetic, algebra, geometry, and trigonometry is classical Greek mathematics", "the work of Greek geniuses." This, in spite of the fact that the Greeks had no positional numeration system (Indian in origin) and that algebra, as the name indicates, has a (mostly) medieval Arabic origin. Greek mathematics is certainly imposing in its theoretical organization, amount, and complexity, but the computational cast to early modern mathematics and science owes far more to other cultures than it does to the Greeks.

One can point to numerous deficiencies in the specific accounts given for each culture. Let me point out a few shortcomings for some different cultures.

Goodman discusses Egyptian computation and notes its restrictive use of unit fractions, even showing a method for converting various common fractions into their unit-fraction-sum equivalents. But nowhere does he do any genuine arithmetic with fractions or even mention the central computational significance of the 2-divided-by-n results for odd n up to 101. When a problem involving fractions needs to be solved, he simply resorts to modern algebraic manipulation and fraction calculations.

Goodman's treatment of Babylonian algebra is also flawed. For the last twenty-five years or so, now, we have known, thanks to the pioneering work of Jens Høyrup, that Babylonian algebra almost certainly has a geometric substrate that grounds the procedures used in solving a wide range of quadratic problems. Completing-the-square algorithmic computations were evidently accompanied by dynamic cut-and-paste operations that actually completed a square diagram. None of this is communicated by the text. Instead, the reader is offered symbolic equations that parallel the computational algorithm and a quadratic formula of sorts in the end. The reader is assured, however, that their procedure is not "some wild manipulation that just happened to luckily give us the right answer, [because] in fact the Babylonian method has a lot in common with our modern technique."

Various aspects of ancient Chinese mathematics are explored in the text. Chapters 7 and 8 in *Nine Chapters on the Mathematical Art* treat problems solved by the Chinese method of excess-and-deficit (double false position) and array calculation procedures (matrix elimination techniques), respectively. These topics are taken up by Goodman, but really only for the problem types they put forward. Chinese solution methods are ignored. The text solves the resulting linear systems of symbolic equations by (inefficiently) substituting expressions for unknowns to eliminate them and so reduce the number of equations; the rationale given for this cumbersome procedure is, ironically, that "matrix algebra wasn't invented when the Chinese came up with [such problems]"! Consequently, there is also no occasion for Goodman to compare the two different Chinese solution methods, to compare the Chinese method to Gaussian elimination, nor to note how negative numbers enter the scene when using the more advanced array method.

Let me close with one example drawn from Greek mathematics. Goodman acknowledges Archimedes' seminal importance for developing a deeper understanding of the circumference and area of a circle. As he puts it, "Archimedes calculated π

better than anyone had ever done before", though he doesn't tell us how he calculated this nor even which π this was, the one connected to circumference or the one connected to area. He also fails to explain that Archimedes' results (contained in his fragmentary work *Measurement of a Circle*) establish why the same constant comes up in both projects. Without saying how Archimedes knew it or even that he approximated the ratio of the circle's circumference to its diameter by using inscribed and circumscribed regular polygons (*Proposition* 3), he claims that Archimedes previously showed that the circumference of the circle was $C=2\pi r$. Archimedes then used this, according to Goodman, to determine the circle's area by cutting it into infinitely many sectors and rearranging them into a stacked rectangle of dimensions $C/2$ by r, giving $A=\pi r^2$. A finitary version of this argument, using a bumpy rectangle, is certainly a good heuristic way to convince elementary students that the area of the circle is intimately connected with its circumference, but so far as I know, this method was never entertained by Archimedes. In fact, prior to approximating the circumference-to-diameter ratio for the circle, Archimedes showed (*Proposition* 1), by a classic double *reductio ad absurdum* argument, that the area of a circle is the same as that of a right triangle whose sides are equal to the circle's radius and circumference, thus reducing the area problem for the circle to its circumference problem: $A=\frac{1}{2}Cr$

. The circumference-to-diameter ratio approximation two propositions later was as close as Archimedes ever got to finding exact values for either the circumference or the area of a circle.

It is obvious from reading *An Introduction to the Early Development of Mathematics* that Goodman has real enthusiasm for teaching mathematics and for telling his students stories about how mathematics developed in a variety of cultures over time. We could use more mathematics texts born of such a desire. Wiley also seems to have taken pains with the book's production; I found only one major typo (a reference to Aristotle on page xiv, where Archimedes was obviously meant). Unfortunately, though, this book did not receive the same sort of scrutiny from someone with expertise in the history of mathematics. Consequently, while some may wish to use this book as a text for a liberal arts math course, I would not consider it suitable for a course on the history of mathematics, on any level, particularly for mathematics education students.