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Mathematization and Modern Science

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The discipline of mathematics has not been spared the sweeping critique of postmodernism. Is mathematical theory true for all time, or are mathematical constructs in fact fallible? This fascinating book examines the tensions that have arisen between modern and postmodern views of mathematics, explores alternative theories of mathematical truth, explains why the issues are important, and shows how a Christian perspective makes a difference.

This chapter continues the process of tracing Western mathematization.

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Comments
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For additional material authored by Dr Jongsma in the same book, see Chapter 5, *Mathematization in the Pre-Modern Period* (pages 133-161) and the first third of Chapter 7, *The Mathematization of Culture* (pages 193-201).

CHAPTER 6

Mathematization and Modern Science

Introduction

Let us take stock of where we are in the process of tracing Western mathematization. Some ancient Greek philosophers had abstractly asserted the preeminence of mathematics for understanding the nature and structure of reality. Other Greeks promoted mathematics more concretely by developing mathematical theories of natural phenomena such as Archimedes' treatment of the law of the lever. However, much of what we now call natural science remained unaltered in any essential way by mathematics. Various writers in the medieval period continued the rhetoric of mathematization, but they provided little solid evidence to back it up. Later thinkers instead followed Aristotle, emphasizing the importance of logical argumentation for natural philosophy and viewing each science as having its own (non-mathematical) subject matter. Mathematics was a field to imitate on account of its deductive method, not one to apply because of its subject matter. An exception to this occurred near the end of the Middle Ages, when the Oxford Calculators developed a mathematical theory of motion, but this remained tied to abstract philosophy.

Renaissance thinkers valued mathematics highly on account of its logical structure and the certainty of its conclusions, but they also put it to work in practical and artistic affairs. While they still thought largely in Aristotelian terms about natural phenomena, habits of measurement and quantification were becoming ingrained in everyday life and provided a platform for seventeenth-century developments in science. As Greek knowledge was be-

ing reassimilated during the Renaissance, old ideas were also being challenged, in part due to discoveries made on voyages to new lands. New ideas and techniques were arising in astronomy, algebra, and computational arithmetic, often in connection with projects that sought to restore mathematics to its ancient glory. Activities that lay at the juncture of practical affairs and mathematical science, such as architecture, engineering, and gunnery, also made advances in the sixteenth century. Furthermore, the Renaissance saw the rebirth and spread of philosophies that asserted the supremacy of mathematics. Since mathematics had proved itself so well on a human scale in Renaissance culture, expanding its applicability to cosmic proportions seemed reasonable. While neither Platonism nor Hermetic philosophy was overly disposed toward an experimental or mechanical approach to natural philosophy, they did contribute to an intellectual climate that challenged Aristotle and promoted the mathematization of science.

Aristotelian natural philosophy came under fire on a philosophical level in the seventeenth century, but that was not the most important showdown. It was the successful outworking of a quantitative outlook within astronomy and mechanics that finally ended Aristotle's domination of natural philosophy. The immense success of mathematized science throughout the century encouraged people to push the program into other areas of natural science and even into the human and social sciences.

Seventeenth-Century Mathematization of Natural Science

The seventeenth century produced unprecedented advances in mathematization. During this period mathematics was harnessed to a mechanistic program of natural philosophy. Science began to be mathematized in a way that anchored it in experienced measurable behavior of natural phenomena.[1] This development began in the mathematical sciences of astronomy and mechanics, where quantification already held a strong position and was even sanctioned by a modern version of Aristotelian natural philosophy that up-

1. Peter Dear traces various sixteenth- and seventeenth-century developments, linking quantification with different notions of experience and experimentation. See his *Discipline and Experience: The Mathematical Way in the Scientific Revolution* (Chicago: University of Chicago Press, 1995).

held the legitimacy of mixing science and mathematics. However, the newly discovered work of Archimedes in mathematical physics was the crucial factor driving this development. It gave scientists an important model and some essential tools for pursuing the project.

The two most important mathematicians who battled the traditional physics and astronomy of Aristotle and Ptolemy were Kepler (1571-1630) and Galileo (1564-1642). In different ways and independently of one another, they attempted to establish Copernicus' view of the universe, both with their scientific findings and their defense of its reasonableness. In Galileo's case, this was part of a broader attack on Aristotelian natural philosophy. He also developed a new mathematical science of motion that contradicted Aristotle's views. Kepler's main work was in astronomy, but he also made an important contribution to the field of ocular optics. Both of them were strong proponents of a mathematical viewpoint on nature; both of them made lasting contributions to mathematical physics. Thus we begin our story of the greatest advances of mathematization with their work.

Kepler's Mathematical Vision of the World

Kepler studied nature to gain a deeper understanding of its Creator. Kepler's success in revealing the secrets of God's magnificent work was offered back to him with exuberant praise and devotion. In an era of religious discord, Kepler was a Lutheran with sympathies toward both Calvinist and Roman Catholic positions on certain theological issues. What was most important for his scientific work, however, was his view of God as Creator. Here his ideas derive primarily from Neoplatonic/Neopythagorean philosophy. For Kepler, God is the Supreme Architect, the one who created the universe according to eternal geometric patterns located in his mind. As Kepler notes:

> Geometry, being part of the divine mind from time immemorial, from before the origin of things, being God Himself (for what is in God that is not God Himself?), has supplied God with the models for the creation of the world.[2]

2. Max Caspar, *Kepler*, revised edition (New York: Dover, 1993), p. 271. This quote appears in Kepler's work of 1619, but it represents his viewpoint throughout his life.

Thus, Kepler believed that God had embodied some of his essential mathematical nature in the creation. Being created in God's image, we humans can think his thoughts after him, using the ideas of number and magnitude he has implanted in our minds. True knowledge of natural phenomena can be attained when the geometric schemes in our minds correspond to those prototypes in the Divine mind that have been copied into the world. Scientific knowledge results from the use of human reason stimulated by experience. Because scriptural revelation does not aim to impart knowledge of nature, it is thus irrelevant to this task. To understand how the universe is regulated and even to learn more about the being of God, we must search out the spatial structure of the world and look for proportion and harmonious relations among its geometric magnitudes.

Kepler's mathematized natural philosophy is clearly seen in his astronomical work. Already as a university student, he became convinced of the correctness of Copernicus' system, being attracted by its overall simplicity and its emphasis on symmetry and harmony. After he became a professor of astronomy, Kepler developed his ideas further. He believed that the five Platonic solids, whose treatment had provided the grand finale to Euclid's *Elements,* could explain the planetary orbits. Using a series of inscribed and circumscribed spheres separated by the five nested regular solids, Kepler accounted for the number of the planets and the spacing of their paths along these spheres. God ordered the planets, he claims, to accord with this beautiful geometric arrangement. He explained this theory in his first astronomical work, *Mysterium Cosmographicum* (1596). Although existing astronomical data failed to support his hypothesis completely, Kepler gave reasons, including possible inaccuracies of the data, for why his theory might not quite agree with tabulated values. Kepler came back to this Platonic vision of the celestial array throughout his life, unwilling to toss it aside. He supplemented it in his third astronomical work, *Harmonice Mundi* (1619), with various other arcane mathematical ideas, such as the identification of musical harmonies with planetary motions. This gave wondrous mathematical detail to the Pythagorean "music of the spheres." Kepler discovered one of these harmonious mathematical relations while trying to find a numerical relationship to demonstrate the Sun's role in propelling planets about their orbits. This result is now known as Kepler's third law: the squares of the periods of the revolutions of the planets are proportional to the cubes of their mean distances from the Sun.

Aspects of Kepler's thought seem to mesh quite well with the blend of

Platonic and Hermetic philosophies of his time. Kepler was initially attracted to an animistic view of planetary motion — that is, that planets were either beings that had souls or were guided by such beings. However, he soon abandoned it for a purely mechanical outlook, comparing astronomical motions to a well-regulated clock. Kepler's use of mathematics also differed from his more mystical counterparts. Kepler pointed this out in a critique he gave of a leading Hermetic thinker who had accused his astronomy of not delving into the inner realities of nature. One must always subject mathematical conjectures to empirical testing, Kepler asserted, and not create mathematical fantasies. We are bound to the world God made and are not free to create one of our own. Kepler thus disassociated genuine mathematical science from numerological speculations and delineated proper uses for mathematics in opposition to the excesses around him. Astronomy can use mathematical hypotheses that accord with appearances, but these hypotheses must do even more; they must demonstrate the way things actually work. The harmonious geometrical structure of reality was an *a priori* given for Kepler, but the exact form of the mathematical regularities it exhibited must be determined from the facts of experience.

This attitude is aptly demonstrated in Kepler's second astronomical work of 1609, *Astronomia Nova.* Ten years earlier he had been hired as Tycho Brahe's mathematical assistant, doing the astronomical calculations associated with Tycho's program of observations. Kepler took on the task of calculating Mars' orbit. Given the precise data for which Tycho was known, Kepler discovered that the accepted ideas about Mars' motion failed to generate accurate values. Continuing to work on the problem after Tycho's death in 1601, Kepler eventually found a solution. It was a solution unlike any he or anyone else had anticipated, however. After years of struggling with computational and conceptual difficulties, he finally concluded that the orbit of Mars was an ellipse. This is an instance of Kepler's first law. The compound circular motions that had been postulated by others to describe each planet's apparent motion were now replaced by a single, simple, elliptical motion around the Sun, which was located at one focus. In addition to relinquishing circular motion, Kepler found that he also had to give up uniform velocity. Mars moves with uniform velocity only in the sense that its radius from the Sun sweeps out equal areas in equal times. This is Kepler's second law. Finding these results took about eight years of patient calculation with empirical data and involved numerous dead ends and mistakes. However, Kepler's new astronomy swept away two thousand years of false

preconceptions and completed Copernicus' system of the world in a surprising and beautiful way.

In addition to discovering these two laws, which he generalized to all the planets, Kepler insisted that the motion of heavenly bodies required explanation. Traditional natural philosophy kept heaven and earth separate and saw no need to stipulate a cause for uniform circular celestial motion. Kepler challenged this. He proposed, for the first time, that physics be combined with mathematical astronomy. He was unable, however, to give a fully acceptable quantitative explanation for planetary motion (following Gilbert, he suggested that a rotating Sun pulls the planets around using some sort of magnetic force). The final outworking of this view had to await Newton's theory of universal gravitation toward the end of the century. Even so, Kepler's astronomy initiated the idea that the world is a mechanical universe in which the planets move due to the physical action of the Sun and in obedience to mathematical laws; they no longer move through the action and will of quasi-divine beings inhabiting the planets.

Kepler's astronomical writings contained technical mathematical discourse, overfull descriptions of the meandering process of his scientific discoveries, Neoplatonic philosophizing, and ecstatic religious utterances. This unique combination did not attract many readers. The full importance of his astronomical viewpoint and results were only gradually recognized. Nevertheless, his *Epitome Astronomiae Copernicanae* (1621) provided the main source for later scientists, such as Halley and Newton, to learn the basic details of the Copernican system he had completed.

Galileo's Mathematical View of Nature

Galileo's *Dialogue Concerning the Two Chief World Systems* (1632) was intended as a popular account of the Copernican system, and it was well known and accepted by many, even though the Roman Catholic Church quickly moved to ban it. In this book, Galileo presented a witty and engaging discussion of the merits of the Copernican system over against the entrenched coalition of Aristotelian cosmology and Ptolemaic astronomy. He showed that the usual physical arguments against Copernican astronomy, arguments directed against the motion of the Earth, are not decisive. The reason objects do not fly off a whirling world, for instance, is because they also participate in the Earth's motion, just as a weight dropped from the

mast of a boat will fall at its base, even if the boat is moving. Galileo defused the standard criticisms with striking and sometimes humorous counter-examples dealing with terrestrial motion, a field in which Galileo was very much at home. In addition to defending Copernicus against his critics, Galileo presented his account of the tides as strong positive evidence for the correctness of the Copernican system. Consequently, he thought Copernicus' astronomy approached the requirements set up by Aristotle for being a demonstrative science. This posture got Galileo in trouble with the Church. He had been given permission to present Copernicus' system of astronomy in the conventional way, as a hypothetical mathematical theory, not as the true system of the world.

Galileo had adopted a Copernican outlook already in 1609. Investigating the heavens with a newly invented telescope of his own construction, he noted the following: the Moon's surface appears to be irregularly shaped, not perfectly spherical; the Sun shows spots that change; Venus goes through phases like the Moon, indicating its revolution about the Sun; and Jupiter has four moons that circle it, rather than the Earth. These discoveries were consistent with and lent support to the Copernican viewpoint, while they presented real problems for the traditional approach. Galileo's report of his findings and of the instrument he used generated far more excitement about astronomical possibilities than Kepler's work and cast him in the role of Copernicus' defender.

In the *Dialogue,* Galileo took a non-technical qualitative account of the Copernican system as the basis for his discussion — circles, uniform motion, and all. He did not advance the Copernican viewpoint using quantitative means, except indirectly by advocating combining physics and astronomy, thereby making physics more mathematical. In fact, while Galileo was aware of Kepler's earlier astronomical work, there is no indication that he ever adopted its results. His interest was more in the physical aspects of the situation and with what he could contribute to the discussion from his knowledge of mechanics. Galileo did promote a highly mathematical approach to physics in this latter field, however.

Like Kepler, Galileo's mathematical viewpoint of the world was grounded in his religious and philosophical orientation. Galileo took a traditional Augustinian viewpoint on the source of natural and scriptural revelation — God is the acknowledged Author of both the book of Scripture and the book of Nature. Therefore, divine truths revealed by either one cannot contradict those of the other, though human interpretations of each might

give rise to conflicts. However, he proceeded to elevate knowledge of nature. Aristotle had said that demonstrative scientific reasoning based on sensory experience provides necessary and certain knowledge of the world and not mere opinion. Galileo agreed. Hence for him, results in this realm should be given primacy and accepted by everyone, including those engaged in interpreting Scripture. The natural light of reason must be granted a higher priority than faith in the realm of natural knowledge. Given the Church's attitude toward lay interpretations of Scripture in the wake of the Protestant Reformation, however, Galileo knew that he had to tread very carefully in this matter.

What makes the knowledge of nature so certain? Here Galileo, like Kepler, proffered a mathematized view of reality whose philosophical roots go back to Pythagoras, Democritus, and Plato. The deductive structure of geometry guarantees the certainty of its results and the use of quantity gives it precision. Nevertheless, Galileo, like Kepler, was critical of the mystical side of Neoplatonism. Natural philosophy is not fiction. In a work of 1623, Galileo explained his overarching viewpoint on science with the following well-known words:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[3]

Following Plato and the atomists and in opposition to Aristotle and scholastic philosophers, Galileo asserted that the essential and necessary properties of material things are the primary mathematical qualities of number, size, shape, and speed. Sensory properties such as color or sound are secondary qualities and reside in the sensing subject, not in the object itself. Galileo thus recognized the foundational role of mathematics for science, but he went further and reduced the central content of physics to geometry. His views on primary and secondary qualities were echoed else-

3. Stillman Drake, *Discoveries and Opinions of Galileo* (New York: Doubleday Anchor Books, 1957), pp. 237-38.

where in Europe, though they likely arose independently, from the same ancient sources that fed Galileo's viewpoint.

Galileo accepted Aristotle's deductive methodology of science,[4] but he gave larger roles to experience and especially to mathematics. Galileo was especially indebted to Archimedes for his approach to mathematical physics. Galileo's outlook in this respect is not unique, for as we noted above, earlier natural philosophers were moving in this direction, similarly taking inspiration from Archimedes' practice. Yet, Galileo was more thoroughgoing in his mathematization and ingenious in his empirical and mathematical exploration of phenomena. His familiarity with the Renaissance tradition of applying mathematics to practical technology may have helped him introduce measurements into physics. He rejected the ancient tendency to see qualities as pairs of opposites, such as hot and cold or being at rest and being in motion, and instead treated them as measurable quantities lying along a continuum. They are thus quantities that can be represented by line segments. In Galileo's hands, natural science was being transformed into a field of thought conducted mainly by means of the ideas and methods of mathematics rather than those of syllogistic logic. Galileo's mathematical notions were not mystical ideas floating high above the field of physics, but were linked concretely to the way things actually function. Experimental exploration was used by Galileo to suggest appropriate first principles and to eliminate or confirm hypotheses, but the core of any true science was down-to-earth mathematics suited for the task. An Aristotelian concept of science was combined with an Archimedean viewpoint on the importance of geometry and proportion, subject to experimental verification.

Galileo is best known in scientific circles for his mathematical analysis of motion. This was published in 1638 as the central part of his *Two New Sciences,* though his initial thoughts on motion go back nearly 50 years earlier. In this, his last work, Galileo presented a science of motion in three parts, first dealing with uniform motion, then with naturally accelerated motion, and finally with projectile motion. Each section is organized in standard Euclidean style, opening with the relevant definitions, axioms, and postulates, and then proceeding to various propositions. For uniform motion, Galileo proves several results relating speeds, times, and distances traversed, using

4. See William A. Wallace, *Galileo's Logic of Discovery and Proof,* Boston Studies in the Philosophy of Science, vol. 137 (Kluwer Academic Publishers, 1992).

the Archimedean tools of ratio and proportionality, ending with the equivalent of our result that distance equals speed times time.

Regarding naturally accelerated motion due to free fall, Galileo first notes that free fall yields uniformly accelerated motion since the results derived on that basis match those gotten by his experimentation with inclined planes. However, he also says this is to be expected since Nature always acts in the simplest way, and the simplest type of accelerated motion is that in which equal increments of speed are added on in equal increments of time. The conclusions he obtains from this agree with those gotten earlier by the Oxford Calculators (whose work he seems not to have depended upon), but they contradicted the commonly held notion that *speed* is proportional to the *distance* fallen. Instead, Galileo shows that *distance* is traversed as the square of the *time* elapsed.

In the last part of the treatise, Galileo determined the path of a projectile. Mathematicians earlier conjectured that the path consisted of three parts, a first and last straight path connected by a curved path, possibly circular. Building on his results about uniform horizontal motion and naturally accelerated vertical motion and assuming that these motions maintained their independence when combined, Galileo was able to establish the exact shape of the path as parabolic. Galileo understood the benefit this gave to military art, for it enabled gunners to compile charts relating the range of their cannon shots to the elevation chosen. Galileo was also able to prove that the maximum range of a projectile would be achieved at half a right angle.

Galileo proudly and rightly advertised his work on motion as an important new science. He had individually done for motion what others had done for optical phenomena and statics — he had created a mathematical science. Traditional natural philosophy, drawing on Aristotle's ideas of motion, treated motion in a mostly qualitative fashion. On those points where it made quantitative assertions, it was often spectacularly wrong. The work of the Oxford Calculators was an exception to this, but their work was abstract mathematics, not physics. Galileo's was both physics and mathematics, treating natural (if idealized) motion. He did not focus on the cause of motion or try to explain how motion could continue, as earlier philosophers had. Instead, he explored its mathematical features and established precise functional dependencies, using the language of proportionality. The question addressed, using mathematics, was not *why* but *how*. This mathematical model was to inspire other scientists in the seventeenth century and was to become

the *modus operandi* of physical science. The significance of Galileo's specific accomplishment is further put into perspective by considering the importance assigned to motion in seventeenth-century mechanistic thought: along with the mathematical features of number, size, and shape, motion was thought to lie at the base of all other natural phenomena. Galileo provided the mathematical foundation and some of the tools for developing this outlook. Newton was to take Galileo's work and incorporate it into his own later in the century, using a methodology that owed much to Galileo's approach.

Descartes' Mechanistic Mathematical Universe

Mechanistic explanations were becoming the new wave of natural philosophy. Aristotelian natural philosophy had been modeled on living organisms and emphasized teleology, a notion of purposeful development. Now, scientists were beginning to analyze phenomena largely in terms of their mathematical relations and mechanical behavior. Complex automata and intricate machines became the new model. Kepler and Galileo were far from being the only scientists to adopt such an approach. Early in the century, Isaac Beeckman, drawing upon a practical mechanical tradition in the Netherlands as well as his knowledge of Archimedes' theoretical works, was one of the first to formulate a mechanistic outlook. Since quantitative features of natural reality are all-important, he says, mathematics supplies the hands of physics. These ideas exerted a lasting influence on seventeenth-century thought through the medium of Descartes (1596-1650), who learned this mechanistic approach first-hand from Beeckman in 1618-19.

Descartes was neither an astronomer like Kepler nor a physicist like Galileo, though he contributed to both fields. He was primarily a systematic philosopher, whose work in mathematics was also of the first magnitude. The methodology and subject matter of mathematics, Descartes believed, holds the key to natural philosophy. His goal was to argue this for all natural phenomena, not merely astronomy, mechanics, or optics. Focusing on shape, size, and motion advances the study of nature, for the core meaning of matter and its primary quality is extension. Physics is nothing more than geometry. Thus natural philosophy can be placed upon an indubitable foundation, for mathematics is the paradigm of irrefutable knowledge. Galileo, Descartes notes in a 1638 letter to Mersenne,

> philosophizes much better than the usual lot, for he . . . strives to examine physical matters with mathematical reasons. In this I am completely in agreement with him, and I hold that there is no other way of finding the truth. But I see a serious deficiency in his . . . [not] considering the first causes of nature. . . .[5]

According to Descartes, Galileo should have been more systematic and built his science on a firmer metaphysical foundation. Descartes aimed to develop a method that would solve specific scientific problems but that would also give absolutely certain demonstrations of the true structure of all of reality. In this respect his overall object and view of science was much the same as that of Aristotle, though his mathematized mechanistic approach was very different.

Descartes had hoped to demonstrate the veracity of the Copernican system of the world. However, when the Roman Catholic Church condemned Galileo's ideas, he sought to present his own system in a way that would not collide with the censors. This could be done, he thought, by grounding it in a metaphysics that was consistent with Catholic theology. Beginning with an attitude of radical doubt (in order to counteract even the extreme skeptic), Descartes notes that doubting requires a thinking subject. This in turn requires a trustworthy Supreme Being who can guarantee our existence and the truth of what we know. God alone can legitimize the nature and certainty of human reasoning. We humans have been given the capacity by God to generate clear and distinct ideas of things by the operation of our minds. These innate ideas provide us with mathematical and other notions for understanding our world.

Descartes thus proposed a strong rationalist philosophy. There are two irreducible types of reality, according to Descartes: mental and corporeal. Matter behaves in a completely mechanical fashion; mind comes to know how matter acts by means of its clear and distinct ideas. Humans alone have reason and are essentially thinking beings. All other living things are really nothing more than automata. We can apply our reason to attain true knowledge of the natural world from first principles. At times, however, especially given the incomplete state of our knowledge, we may have to resort to hypothetical reasoning, as was done earlier in astronomy. Then we must check the

5. William R. Shea, *The Magic of Numbers and Motion: The Scientific Career of René Descartes* (Cambridge: Science History Publications, 1991), p. 312.

deductive consequences of our hypotheses with experience to verify or falsify them. Experience provides the data requiring explanation, and it helps us to decide how to generate an explanation for them from *a priori* principles. While Descartes did quite a bit of experimentation in connection with optics and physiology, its role in his thought was mainly that of determining what Reason needs to render an explanation for and of suggesting some possible connections. True science is ultimately generated, however, by drawing necessary conclusions from self-evident axioms known *a priori* by our minds.

Descartes stressed the essential importance of method in acquiring knowledge. The general approach that Descartes saw embodied in the mathematical method of analysis can also be called the method of philosophical analysis. He exhorted anyone who wanted to advance in natural philosophy to consider all relevant phenomena, analyzing and clarifying the various notions involved into more basic constituent concepts, until arriving at the simplest ideas. These ideas can then be taken as foundational, more complex ideas being built up from them, until a theoretical explanation of the phenomena is obtained. It was this rationalist approach of breaking reality down, as one might a machine, and then reconstructing it from its abstracted elements, building knowledge upon a basis of clear and distinct ideas that attracted his followers. However, the metaphysical underpinnings for this approach (first doubt, then God, then clear ideas), which Descartes saw as necessary, was of less interest to many. They were captivated by Descartes' vision of a mechanical universe susceptible to mathematical analysis, but not always by his philosophical rationale.

The value of method was certainly not a new theme in philosophy, but Descartes was the one who made it fashionable for the modern age, drawing upon his work in mathematics. Mathematics contains not only the time-honored deductive mode for communicating known truths, but even more importantly, an analytic method for discovering truths. He published his ideas on this in *Discourse on Method* (1637), to which he attached three appendices as proof of the value of his approach: an essay on optics, another on meteorology, and a concluding one on geometry. Like Viète, Descartes found vestiges of the analytic method in Arabic and Renaissance algebra and in the geometry of Pappus and other ancient Greeks. Unlike Viète, however, he maintained that his algebra and analytic geometry were more advanced than those of the ancients, for he was able to solve problems that had defied their best efforts. In his essay on geometry, we can clearly see our modern

approach to symbolic algebra and analytic geometry taking shape. Descartes' influence in this area is apparent from the similarity of his work to what we know these fields to be, though the presentation is still somewhat obscure in spots. In his appendix on optics, Descartes presented his law of refraction. Others in the same period had also come to a correct understanding of this law, but it was Descartes who publicized it and attempted mechanical explanations of it. He also offered an exemplary analysis of the appearance of the rainbow, explaining it in terms of the Sun's multiple refraction through raindrops. Descartes' *Discourse,* with its famous appendices, rightly established his reputation in mathematics, physics, and philosophy as an intellectual giant.

Descartes pursued his mechanistic natural philosophy in all realms, including mechanics, astronomy, optics, and physiology. He proposed a universe completely full of particles, whose collisions with one another produced motion in accord with a number of quantitative rules, some of which anticipate Newton's laws of motion but most of which we no longer accept as valid. The motion of the planets about the Sun was explained in terms of whirlpool or vortex motion of particles in the intervening space. Natural philosophers after Descartes tried to determine the precise mathematics of this motion, without success. Newton later discredited Cartesian physical astronomy by showing that the planets' motions would require contradictory behaviors from the vortex, but his refutation was not accepted by everyone as sufficient reason for rejecting the general idea. In optics, Descartes explained how light travels, using various mechanical analogies, one of them being the instantaneous transmission of pressure through the visual medium. These are then used to explain reflection and refraction. Following the lead of Kepler in his mathematical analysis of retinal images on the back of the eye early in the century, Descartes explicitly separated the physiological aspects of vision from perceptual cognition and treated them completely mechanically.

Descartes' system of natural philosophy may have lacked mathematical specificity in some areas, but on many fronts it initiated a very attractive program to be developed. Descartes showed for the first time the potential of taking a mechanistic approach to all of nature, of treating the material universe as a machine. In the last half of the seventeenth century and on into the eighteenth, Descartes' system of thought commanded the highest respect and formed the basis of further scientific work. After Descartes, Aristotelian natural philosophy was no longer a viable system; mechanistic natural phi-

losophy had taken its place, with Descartes' ideas in the forefront. It was thought by some that Descartes had laid down the basic outlines of the true system of the world and that all that remained for others to do was to fill in the details.

Newton's Mathematical Treatment of Physics

Like every other natural philosopher of the late seventeenth century, Newton (1643-1727) was weaned on Descartes. He came, however, to reject Descartes' thought because of its speculative rationalistic character. For Newton was also nurtured on the ideas of his countryman Francis Bacon, who emphasized the need to generalize from firmly established facts.

Bacon, who wrote in the first quarter of the seventeenth century, stressed detailed empirical investigation of reality, leading to classification and determination of the true natures of things. His approach was not aimed at formulating mathematical laws. Unlike his earlier namesake, Bacon did not recognize the value of mathematics for science. He thought that mathematics' habit of abstraction was dangerous to physics, which needed to remain in close contact with reality. Robert Boyle, Bacon's disciple living in the Cartesian second half of the century, had a much stronger appreciation for mathematics as an aid to science and as a necessary part of mechanical philosophy than his mentor did. However, he still failed to connect mathematics very closely with experimental practice. He believed that mathematics was too independent of physical reality and that its fastidious precision was out of place in the laboratory.

For many at this time, contributions of mathematics to natural philosophy were largely of two sorts, either sublime speculations or mundane observations, but these were not often linked in any intimate way. Mathematics was admitted to be an essential component of mechanical philosophy, since the primary subvisible properties of things that determined their structure and behavior were taken to be quantitative. However, exactly how these microscopic mathematical features determined macroscopic behavior was difficult, if not impossible, to ascertain. Mechanistic natural philosophy thus led to imaginative non-verifiable conjectures. At the same time, more aspects of natural phenomena were being quantified and measured more precisely in this period (time, distance, speed, weight, volume, air pressure, temperature, volume, intensity of light, etc.). This gave a potential foundation

for mathematizing various fields of thought and activity. Yet, these were rarely connected with any mathematical theory about how things worked. The habit of looking for quantitative functional dependencies was still weak; most scientists continued to look for underlying causes in the presumed nature of things. Moreover, many seventeenth-century scientists worked largely in the qualitative experimental areas of natural philosophy, such as chemistry and natural history.[6]

All of this changed with the arrival of Isaac Newton, whose work in the classical sciences of optics, mechanics, and astronomy made deep connections between quantitative features of phenomena and the way the universe worked.[7] Newton, more than anyone else in the seventeenth century, was able to forge an alliance between experimental research and mathematical analysis and so realize the enormous benefits of mathematizing natural philosophy. He synthesized earlier work in the mixed sciences and laid a foundation for eighteenth-century developments in various other physical sciences.

Newton first demonstrated his brilliance with his investigation of colors. Soon after receiving his Bachelor's degree from Cambridge in 1665, Newton undertook a series of optical experiments to determine the nature and behavior of colored light. Rather than holding that colors are mixtures of light and dark, modifications of white light, as most held, Newton asserted the opposite, that white light is a mixture of different colors. Light can be spread out into a spectrum by a prism because the rays of each color are refracted according to its own characteristic degree of refrangibility. Colors are primary for Newton; white light is the mixture. He established this revolutionary viewpoint by a series of carefully controlled experiments using various prisms, making the appropriate precise measurements and calculations to test his ideas against those of others, such as Boyle and Hooke, and to forestall their criticisms.

He summarized his work on colors in his first paper presented to the Royal Society (1672). In addition to describing the experimental basis for his results and organizing his conclusions in the deductive fashion of mathematics, Newton advanced a new corpuscular theory of light. It was this aspect of his work that received the strongest criticism by others, particularly

6. At the time, the fields that we today call biology, geology, anthropology, etc. did not exist. "Natural history" was their predecessor. Its approach was primarily descriptive.

7. Literature on Newton's scientific work is an entire industry. See, for instance, Richard S. Westfall, *Never at Rest: A Biography of Isaac Newton* (Cambridge: Cambridge University Press, 1980).

Hooke. The unpleasantness of this experience made Newton wary of including hypothetical elements in his scientific theories, and it forced him to try to develop a method of philosophical investigation that would prompt rational acceptance rather than dissent. In response to his critics, Newton asserted that in the main, he had investigated and positively established various properties of light by means of experiments, mathematically deducing their consequences. Only once this was accomplished, did he put forward a conjecture to explain the behavior. This, he noted, was the proper scientific method for doing physics.

Newton elaborated his method of philosophical inquiry in various later works as well. Experiments must be designed to elicit answers to specific scientific questions; thus regularities of observed phenomena are identified and measured. Once general principles are arrived at by induction, their consequences can be deduced by rigorous mathematical demonstrations. Additional experiments verify these consequences as a check on the principles. While this process may not generate absolutely certain knowledge, it gets as close to it as the subject matter permits. Newton's belief on this score conflicted with many of his peers who had come to believe that empirically based knowledge must remain fairly tentative, and also with those who were content to argue from hypothetical mechanical causes. Newton believed that true knowledge could be generated in physical science by applying the method and ideas of mathematics to the data of experience.

After one ascertained the mathematical behavior of phenomena, one might then attempt to discover the causes that produce such phenomena. Newton, like both Descartes and Aristotle, desired to penetrate to the true causes underlying the behavior of reality. Unlike Aristotle, however, he remained far longer on the level of phenomenological behavior and gave priority to quantitative features of the situation. Unlike Descartes and his followers, he was unwilling to use hypotheses to evade empirical exploration and induction. And, unlike both, he refused to build a grand philosophical system based on ultimate causes to encompass his results. He was content to leave this for posterity to work out, if possible. His mathematical and experimental exploration of more limited areas of thought put Newton more in the tradition of Galileo than Descartes or Aristotle.

Mathematics, for Newton, as for Kepler, Galileo, and Descartes, held the key to natural philosophy. Yet, for Newton it seemed to be more an operational stance than an ontological or epistemological position. Whatever the actual nature of the physical world or the precise character of human know-

ing, he was concerned to concentrate first on the indubitable mathematical regularities nature exhibits. This limited focus was emphasized in the title of his magnum opus, *Philosophiae Naturalis Principia Mathematica* (1687). As Newton noted in the preface: "the whole burden of [natural] philosophy seems to consist in this — from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena." Since motions and forces are quantitative matters, and since demonstrative knowledge of these was sought, Newton used geometry throughout. His book was a masterful exposition devoted solely to mechanics and astronomy, but he held out the possibility that a similar exploration in other realms of physical behavior, such as optics or magnetism or chemistry, would likewise uncover relevant forces operating between particles. This suggestion, given in the context of the marvelous achievements of his work, was to encourage similar work in other fields and foster an even stronger viewpoint on the importance of mathematics than he himself may have entertained. Newton wanted to emphasize a dynamic mathematical approach to natural science in contrast to Descartes' more speculative mechanistic natural philosophy, but he did not think that this exhausted everything that could be said about how the world functioned. Nevertheless, the character of his work did lead to a more constricted view of the methods and scope of natural science.

The *Principia* resulted from about two and a half years of intense intellectual work in which little else engaged Newton's attention, including food and sleep. Like a man possessed, he threw himself into developing a force-based mathematical theory of mechanics and astronomy. To ground his astronomical conclusions, Newton first created both a version of geometry that drew from his earlier work in calculus and a new deductive science of mathematical dynamics that built on and corrected Descartes' mechanics. Newton elaborated these ideas in detail, refined or defined new quantitative concepts such as mass and inertia, and gave precise quantitative formulation to the laws that govern motion. Newton showed in the abstract, for example, using only mathematics, that Kepler's laws imply an inverse square law for a centripetal force attracting two bodies that obey his laws of motion, and conversely that an inverse square law entails Kepler's laws. Consequently, given the physical behavior of the planets summarized by Kepler's laws, the force of gravitational attraction between the planets and the Sun must obey an inverse square law. He also proved that the power of gravity must be proportional to the masses of the bodies involved. While Newton offered no ex-

planation of the nature or cause of gravity, he asserted that the force of gravitational attraction acts uniformly throughout the universe, being able to account for and therefore being responsible for both the movement of heavenly bodies and that of naturally accelerated objects on Earth. It was this mathematical encapsulation of the cause of all natural motion, the law of universal gravitation, that most people found so utterly amazing — that the most diverse movements throughout the universe could be explained by a single rather simple principle was astounding! This achievement convinced even the most skeptical that absolutely certain knowledge of the world might be possible using the tools of mathematics.

Newton's ingenious efforts established him as the leading mathematician and the foremost natural philosopher of Europe. No other work in the history of science compares to the *Principia* in scope or importance. Newton combined, revised, and completed Galileo's science of motion, Descartes' mechanics, and Kepler's mathematical analysis of celestial motion, treating them all in his own new mathematical science of dynamics and astronomy. Notwithstanding this accomplishment, not everyone was ready to accept the intrusion of what seemed to be an occult force acting at a distance (gravity) into natural philosophy. In fact, resistance by Huygens and Leibniz on this point kept Cartesian natural philosophy alive for some time into the eighteenth century. However, the coherence, simplicity, scope, and depth of Newton's system were universally admired and eventually helped to overcome the opposition. Newton had shown how to unify various aspects of physical science in one simple harmonious mathematical system, and his success gave others hope of making additional conquests by the same mathematical and mechanical method. Newton's system of the world set the future course of Western thought and helped to define the modern worldview. More narrowly, it established a new mode of mathematizing natural science. Before Newton there were expert experimentalists and accomplished mathematicians, but few top-rank mathematical experimentalists. Newton's approach gave both experiment and mathematics their due. Now analysis (quantitative experimentation, involving measurements, leading to appropriate inductive generalizations) and synthesis (logical deduction from accepted principles, using mathematical theories, concepts, and techniques designed for the purpose) became full partners in the scientific process. Quantitative measurements were tied to theoretical analysis and related by means of functional dependencies, and mathematical tools were developed to help analyze experiential phenomena. Mathematics and physics were now

fused on a deep level, making it possible to predict behavior of phenomena with great precision and to communicate these results in a universal language of clear and distinct ideas. Newton overestimated the ability of experiment to determine scientific principles, and he underestimated the hypothetical element involved in this process, but his emphasis on grounding scientific work in experimentation and placing restrictions on hypotheses was a necessary antidote to the practices and outlooks of many of his contemporaries.

Newton is also well known as one of the main founders of calculus. This is not the place to discuss his role in the history of calculus, but we will comment briefly on how his calculus fits into the mathematization of science. Newton's notion of fluxion, which he began working with already in 1665, is essentially a time derivative. Treating all variable quantities as magnitudes changing over time (fluents), his method of fluxions enabled him to conceptualize velocity and acceleration as mathematical notions and produce techniques for calculating rates of change when the rule for this change was known. He was also able to reverse the process and find fluents (functions) when their fluxions (derivatives) were known. Having discovered and demonstrated the *Fundamental Theorem of Calculus,* Newton could use these reverse methods for calculating areas, volumes, and arc lengths. None of these things appear in Newton's *Principia,* which was written using the classical language of geometry, but their relevance to the topics it covers should be quite apparent, and later mathematicians in the eighteenth century recast and extended his mechanics using the apparatus of calculus. Natural science deals with dependency and change; calculus is the premier theory of functional change. In this respect, too, Newton contributed in an essential way to the mathematization of physical science.

Leibniz's Mathematization of Thought

The other principal founder of calculus was Gottfried Leibniz (1646-1716). Though he arrived at his ideas about a decade after Newton, he was first to publish (1684) and attracted the two Bernoulli brothers to help him develop them further. Using notions of the sum and difference of infinitesimally small quantities, Leibniz made the differential the centerpiece of his brand of calculus. The first calculus textbook, written by L'Hôpital in 1696, followed this approach. Leibniz's ideas and notation soon became standard in

Germany, France, and Holland, while Newton's were largely stranded on the British Isles. After some initial acrimonious exchanges regarding priority of discovery, each side maintained the superiority of its approach without very much positive interaction with the other side for over a century.

Leibniz's ideas on calculus contributed more to the actual process of mathematization than Newton's, since his version became the tool of choice for eighteenth-century mathematical science, which was principally developed by Continental scientists. However, there is another side to Leibniz's thought that is even more strongly devoted to mathematization. That is his scheme for developing a universal calculus of thought.

Leibniz, like Kepler, Galileo, and Descartes, emphasized the ontological and epistemological value of quantification. Like many others at the end of the seventeenth century, Leibniz found a mechanistic viewpoint of science very appealing and strongly promoted it in his works. The essential properties of things are those which can be quantified: size, shape, position, motion, and force. Without the use of quantitative explanations, no understanding of the world is possible; with it, one can determine why things are the way they are and why they cannot be otherwise. For the philosopher Leibniz, as for his predecessors Aristotle and Descartes, the goal of science was obtaining necessary, demonstrative knowledge about the world. Like Descartes, Leibniz sought to ground natural philosophy in the nature of God through metaphysical argumentation. Leibniz found Newton's work rather deficient on this score; his system of the world would run quite as well without God as with Him.

In order to extend mathematics' success in generating absolutely certain knowledge to other fields of thought, Leibniz advocated the dual process of analysis and synthesis. This theme had its proximate source in the late sixteenth century and echoed throughout the seventeenth century, Descartes being the most important instance. On the surface, Leibniz's description of the dual process sounds quite traditional. Analysis means breaking down each concept/proposition into its prerequisite concepts/antecedent propositions and these into theirs, until finally arriving at the most primitive concepts/first principles. Synthesis means reversing the process and using these foundational results to define/prove the given concepts/propositions. Leibniz believed that this process could be applied in any area of human thought, not just mathematical science. By means of rational analysis, the basic concepts and principles of a given field can be determined, and these can then be combined to yield truths to which all rational beings will have to

give assent. This emphasis on analysis would continue to gain strength into the eighteenth century, especially since analysis was associated with the field of algebra, which had been successfully widened to produce both analytic geometry and, with the work of Leibniz, calculus.

So far, Leibniz has merely amplified Descartes' approach, but he next gave the whole procedure a further twist that revealed his extreme commitment to mathematization. The process of analysis and synthesis, he believed, can be effected in a mathematical way by choosing appropriate symbols for the basic concepts and their combinations and then calculating with them to obtain the consequences of the principles, much as is done in ordinary algebra. Already in his *Dissertation on the Combinatorial Art* (1666), Leibniz gave voice to this vision: "there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, and say to each other: Let us calculate." Eleven years later, after he had begun developing his differential and integral calculus, he reiterated the point, in the following words: "All inquiries that depend on reasoning would be performed . . . by a kind of calculus. . . . And if someone would doubt what I advanced I should say to him: Let us count, sir; and thus by taking to pen and ink, we should soon settle the question."[8] Leibniz's notion of a universal rational calculus that can be used to formulate and solve problems in all areas of human activity may seem an extremely naive form of utopian rationalism, but a similar hope has motivated others in Western culture since the time of Leibniz. Boole's algebraic logic of the mid-nineteenth century, for instance, matched some of Leibniz's ideas almost exactly, though it was developed independently. Aspects of computer science in our own era — the development of expert systems and the strong program of Artificial Intelligence (we will discuss this in detail in Chapter 9) — can also be viewed as intellectual descendants of Leibniz's brainchild.

It is clear that Leibniz is on Descartes' side of the philosophical seesaw pitting reason against experience. Leibniz stresses the value of *a priori* reasoning over experience. He thought that British scientists, such as Boyle, overdid it with all their emphasis on experimentation. The senses are unable to give rise to certain knowledge. Natural science demands more, which only demonstrative reasoning can satisfy. Leibniz's attitude toward a rational

8. Alistair C. Crombie, *Styles of Scientific Thinking in the European Tradition: The History of Argument and Explanation Especially in the Mathematical and Biomedical Sciences and Arts*, 3 vols. (London: Duckworth, 1994), p. 1009.

physics, combined with Descartes' similar sentiments, overpowered Newton's more balanced approach in subsequent developments. Eighteenth-century thinkers expanded the fields of physics opened up by Newton and went into related fields as well, but they did so using the rationalistic approach of Descartes and Leibniz. The lure of being able to attain absolute truth through human reason and mathematics was too strong to resist.

The Ongoing Mathematization of Natural Science

Eighteenth-century mathematics was primarily mixed mathematics. Mathematical research in calculus and differential equations went hand in hand with work in the mathematical sciences of mechanics, astronomy, fluid mechanics, acoustics, and others. There was no such thing at the time as separate fields of pure and applied mathematics, although distinct areas of mathematics did start to separate from physics as the century wore on.

Eighteenth-century mathematics' being tied to science meant that the mixed sciences were still as much a part of mathematics as they were of physical science. This fit the classical approach going back to the Greeks, but in the eighteenth century it was part of the Cartesian legacy, augmented by Leibniz's viewpoint. As we noted above, Newton's own approach, which emphasized experimentation as well as mathematics, was neglected by many eighteenth-century scientists, especially, ironically enough, in the field of mechanics, where a strong *a priori* rationalist approach predominated.

Leading scientists such as d'Alembert (1743) and Lagrange (1788) treated mechanics as a closed mathematical system whose results could be rigorously demonstrated from necessary first principles. This was accompanied by banishing geometry from mechanics and reformulating mechanics in purely analytical terms, sometimes without any diagrams or reference to spatial content. Such a treatment of mechanics was usually coupled with a deterministic philosophy of nature. Mechanics was thus deemed capable of generating absolutely certain knowledge about the ultimate structure of the world. Given the initial conditions of the world system, one could in principle predict the future state of the entire material world at any time. This sort of mathematical determinism was present in Laplace's analytic treatment of astronomy (1799), which showed that Newton's system of the world was even more stable than had been previously thought. Eighteenth-century mathematical scientists also advanced other areas of physical science worked

on by Newton, such as fluid mechanics and acoustics. They attempted to place them on a firmer mathematical basis of self-evident mechanical principles and explore their deductive consequences using techniques of algebra and calculus.

The areas of electricity, magnetism, heat, and chemistry remained largely experimental during the first half of the eighteenth century as scientists were becoming more familiar with the basic phenomena they exhibited. By the end of the century, however, scientists had learned how to define and measure different quantities associated with them, using instruments designed for the task, and in some cases had begun to formulate mathematical theories for them. In 1787 Coulomb published the results of his carefully controlled experiments in static electricity and magnetism, establishing inverse square laws for both types of attraction, just as for gravity. In the first half of the nineteenth century, a deeper analysis of electromagnetic phenomena by Faraday and others led to an impressive unified mathematical treatment of all these fields by Maxwell in 1865, using notions of vector analysis.

Joseph Black was responsible for initiating a quantitative approach to heat. He first distinguished quantity of heat from temperature in 1760, relating the two by means of the notion of specific heat or capacity for heat. He also introduced the notion of latent heat associated with change of state. A fully mathematical treatment of heat, however, had to wait until early in the nineteenth century, when Fourier studied heat diffusion (1822), investigating it with what we now call Fourier series.

The related area of chemistry became more mathematical by quantifying heat, but also through the systematic use of weight, measured by improved balances. The need to make precise metallurgical analyses of ores for mining provided a strong economic impulse for quantitative chemistry by mid-century. Lavoisier's reform of chemistry (1789) made it more mathematical in a couple of senses. It introduced a more systematic way of naming and symbolizing chemical substances, and it attempted (though only with partial success) to use weights and equations to explain chemical reactions. A more thorough and deeper mathematization of chemistry came about when Dalton put forward his atomic theory in 1808. Using his notion of atomic weight and the law of definite proportions formulated by Proust a decade earlier, scientists could now explain chemical reactions quantitatively in a systematic manner.

Developments during the eighteenth and nineteenth centuries thus

made it abundantly clear that the influence of mathematics in natural science was not about to dry up any time soon. Rather than assisting with a few matters in mechanics and then retiring to its own corner, mathematics continued to find significant and essential employment in the physical sciences. The stream of scientific mathematization continued to swell exponentially over time and has grown unabated into our own era, with no sign of let-up. Obviously the mathematical approach of modern science has uncovered genuine and intrinsic connections between natural phenomena and mathematical concepts and techniques. This conclusion seems undeniable. The question that this success often leaves unasked, however, is whether there may be important aspects that have been overlooked due to taking a rather narrow quantitative perspective. And, more importantly, whether the achievements of mathematical science and the elevated view of mathematical truth have given false encouragement to other areas of human life to pursue mathematization where it is less appropriate. We will address some of these matters in the next chapter.

Mathematization of Science and the Modern Worldview

Developments in seventeenth- and eighteenth-century natural science brought about a major change in people's perception of the world. At the beginning of this period the world was still a universe full of inherent purpose and Christian mystery, though a number of people were becoming skeptical about the validity of knowledge provided by Aristotelian philosophy and traditional religion. By the middle of the eighteenth century, however, the world envisioned by Europe's leading thinkers was one of matter in motion operating mechanically according to universal mathematical laws. Mathematical science began to replace scriptural revelation as the acknowledged authority about the nature of the world. Guided by this mechanistic and mathematical outlook, natural philosophy had made remarkable progress in producing certain and reliable knowledge about the physical world.

The role of human agents in such a mechanical universe was unclear and problematic. For some, humans transcended nature on account of their rational faculties. Humans also had free will that enabled them to be more than passive lumps of matter obeying deterministic laws of Nature. Others felt that the methodology of mathematical science should be pushed as far as

possible. Given its grand successes in astronomy, optics, and mechanics, why not adopt a similar approach in analyzing human nature and social behavior? This path seemed to them to hold the potential for finally arriving at incontrovertible objective truths in these areas as well.

Our next chapter examines some of these more radical developments; we will conclude this chapter by first summarizing what we have learned about the mathematization of natural science and then evaluating these developments from a Christian perspective.

Historical Summary: The Mathematization of Modern Science

The dominant role played by mathematics in natural philosophy in the seventeenth and eighteenth centuries is closely connected with its philosophical heritage. The original Pythagorean viewpoint, adopted and modified by Platonic and Neoplatonic philosophies, deified mathematics, raising it to a position of absolute importance for scientific knowledge. Mathematics alone was seen as capable of penetrating the secrets of the universe, of tracking all things back to their lair. This pagan viewpoint went virtually unchallenged by medieval thinkers. It also influenced a number of Renaissance developments. A revival of Neoplatonism along with Pythagorean-like Hermeticism in the late Renaissance encouraged early modern thinkers to elevate mathematics far above the place given it by traditional natural philosophy.

However, more than philosophy was responsible for the exalted position of mathematics. Mathematics also made itself indispensable in the arts and in the arena of practical affairs during the Renaissance. Here a number of very down-to-earth connections between mathematics and reality were established. This assumed a vastly different role for mathematics than that envisioned by mystical mathematical philosophers, but it was one that nevertheless emphasized the essential necessity of mathematics. Recovery of a number of ancient Greek works, both in mathematics proper as well as mathematical science, also gave a boost to mathematization. Toward the beginning of the modern era, the works of Archimedes exerted a strong positive influence by demonstrating just what could be accomplished in mathematical science.

As Kepler, Galileo, and Descartes worked out their philosophical perspectives in mathematical science, their impressive achievements established

a closer working relationship between mathematics and natural philosophy and seemed to validate their mathematized approach. Newton seems to have entertained a more moderate view of mathematics' place in the universe, but his success in determining the mathematical principles of natural philosophy only gave additional momentum to mathematization in science. Advances in mathematics itself also made a strong impression on scientists. Tools were now available for tackling problems that the ancients could not even formulate, much less solve. The magnificent success of mathematics seemed to feed the philosophical perspective that gave it birth, producing a spiral of scientific progress and mathematized scientific philosophy.

By the start of the eighteenth century, mathematical science had transformed natural philosophy in a revolutionary way. The enterprise was no longer what it was at the start of the modern era. The goal of mathematical science was simultaneously more modest and more ambitious than that of natural philosophy. It was more modest in that now the goal of science was restricted to describing mathematically the way natural phenomena function. This involved a twofold reduction: only mathematical features were thought relevant now, and the ultimate nature of things need not be determined, only its behavior. On the other hand, the goal of mathematical science was more ambitious, for it aimed to plumb the very depths of reality with detailed mathematical precision and logical certainty. It was not content to settle for postulating occult causes or constructing a deductive system of hypothetical knowledge, but intended to specify how reality actually works on a deep level.

The success of mathematical science in achieving what was judged to be universally true knowledge of the world made it the envy of all other areas of thought. As we will see in the next chapter, various social sciences and humanities followed suit, using the rational methodology and techniques of mathematical science. In that way they hoped to push back ignorance and rise to the level of science themselves. Uncovering basic laws of human nature and society, they would then be better able to master it and so gain control over human destiny.

Looking back on these developments with historical hindsight, we can see fruitful connections between philosophical perspectives and positive scientific work, but we can also see the limitations and aberrations that this collaboration produced on a broader scale. Science, powered by mathematics, was installed as *the* source of absolutely certain, objective knowledge, or at least, as the source of the very best knowledge that was humanly possible.

Human Reason muscled out Divine Revelation in the end, though it did not start that way. The development of modern science was closely associated with Christianity, and Christians were deeply involved in developing it,[9] but this collaboration was pursued with an insufficiently critical testing of the philosophy that came along with it. As time went on, scientific philosophy first moved to a deistic viewpoint in which God no longer had any lasting role to play in his mechanistic universe. It finally became a naturalistic viewpoint in which God was completely irrelevant, if he even existed.

The Role of Mathematics in the Modern Scientistic Worldview

What part has mathematics played in the development of this secular outlook? Mathematics has contributed centrally to the modern scientistic worldview, through both its content and its methodology. We will look at each of these briefly in turn.

The aspects of reality that are considered important for scientific work in the modern era are the primary qualities associated with quantity: number, size, shape, position, motion, and force. Other aspects of reality are ruled out as irrelevant or reducible to those of mathematics. Scientific measurements generated by experiments provide the numbers upon which the techniques of mathematics operate. Scientific laws stipulate functional dependencies holding between such magnitudes. Mathematical theories explain the lawful regularities of observed phenomena and predict behavior. This role for mathematics holds in the physical sciences; it was extrapolated into other natural, social, and human sciences as well. Today numbers and graphs are used for quantifying anything and everything. Quantifying gives knowledge, and knowledge is power. At the very least, statistics have now invaded every part of our life, presumably giving us an objective basis for rational decision-making.

The methodology of mathematics has also had a great impact upon the development of the modern scientistic worldview. We can distinguish

9. This theme has been developed by a number of works on science and religion, such as Nancy R. Pearcey and Charles B. Thaxton, *The Soul of Science: Christian Faith and Natural Philosophy* (Wheaton, Ill.: Crossway Books, 1994). Our treatment of mathematization in Western culture looks at the relation between Christian thought and science from quite a different perspective.

two main aspects here. On the one hand, the traditional axiomatic method of mathematics contributed its view of truth and consequences to science. The ultimate goal of physical science, according to the modern view, was to determine absolutely certain first principles and then to logically derive all other results in the field from them using the tools of mathematics. In this way, one will arrive at the most certain knowledge possible, given the empirical nature of the subject matter. Different people evaluated the attainability of this goal and the need for experimentation differently, but arriving at certain knowledge remained the goal of science. Over time, it was thought, scientific progress would generate closer and closer approximations to the truth.

On the other hand, the analytic tradition in mathematics revived a more general notion of analysis that became the paradigm for how scientific discovery should proceed in all areas. The period from Descartes through the Enlightenment can aptly be called the Age of Analysis. Mathematics also provided a well-developed science of analysis, which in the end included algebra, analytic geometry, calculus, and differential equations. These areas provided the ideas and techniques that helped physics analyze basic concepts, solve problems, and derive results from given principles or conditions using algorithmic procedures. Mathematical analysis provided the motive power for the scientific revolution.

A Christian Response to Western Mathematization of Science

What might a Christian perspective on mathematics have to say about all of this? First, a Christian perspective on mathematics can acknowledge with appreciation the positive contributions that mathematics has made to the development of natural science. The process of mathematization has uncovered intimate connections between mathematics and science that reveal the marvelous coherence of creation — something for which we can glorify God. Furthermore, pursuing such knowledge is consonant with a Christian vision of human beings as God's stewards of his creation. That is, an enriched mathematical understanding of how the natural world behaves can help us serve God's purposes in various areas of life. Mathematical knowledge helps us fulfill the cultural mandate given to humanity by God in the garden of Eden.

As Christians, we can also give assent to the reality-oriented stance of the modern scientific outlook: the subject matter of mathematics is certainly relevant to our experience of created reality, and vice versa. Mathematics is not a purely human mental construction, even though mathematics obviously involves the rational operations of our own minds — abstraction, generalization, comparison, deduction, etc. A Christian outlook on the nature of mathematical objects agrees with Platonism to this extent: mathematicians discover lawful regularities in conceptual entities whose existence and properties are largely independent of human intellectual activity.

On the other hand, while affirming mathematics as a good gift of God for understanding quantitative aspects of our world, a Christian perspective on mathematics will take exception to those aspects of the modern worldview that arise from the absolutization of mathematics. Mathematization frequently rejects non-mathematical aspects of life as unimportant or nonexistent and so promotes a lopsided vision of reality. Such a reductionistic program denies the validity of the rich variety of aspects within creation that go beyond quantitative properties. Radical mathematization only works when people are willing to narrow down their perspective of what is real, when they accommodate reality to mathematics as well as conversely.[10] As Christians, we should instead hold a more modest view of the nature of mathematics and its accomplishments, and we should welcome other dimensions of reality as complementary to those studied by mathematics. Thus the perspectives of the craftsman fashioning a beautiful object, the dramatist using words to create powerful portrayals of emotions, and the historian who interprets the meaning of past developments, ought to be respected equally with that of mathematics. A Christian perspective will deny the modern claim that mathematics has a corner on the truth about the world, that it is the final arbiter of all meaning. Mathematics cannot penetrate to the very essence of the universe; God is more/other than a supreme mathematician. The world has non-mathematical as well as mathematical structure.

Rejecting mathematical imperialism, a Christian perspective on mathematics allows us to consider alternative visions of how God, humans, and the world are interrelated with respect to mathematics. In the Neoplatonic

10. This point is argued by a number of case studies in Theodore M. Porter, *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life* (Princeton: Princeton University Press, 1995).

outlook, adopted by various seventeenth-century thinkers, humans thought God's mathematical thoughts after him in order to understand the structure of the world, and these thoughts were seen as eternal and essential to God's nature. Such a view tends to deify mathematics, as we have seen. If we do not equate mathematics with necessary knowledge that is true of all possible worlds, we can entertain other ideas about how mathematical knowledge arises in human experience. We may not accept the postmodern alternative to modernism on this point, but we are certainly freed to explore other options than Neoplatonic rationalism.

Finally, a Christian perspective will view human beings more as integral parts of creation than what is envisioned by dualistic, rationalistic philosophy. Humans are not rational agents set over against the rest of reality; we are not thinking beings situated in a material world that possesses only primary quantitative features. Nor are we masters of our destiny or in control of science and culture solely because of our mathematical scientific abilities. We are creatures of the Lord, meant to exercise our analytical and quantitative abilities in the service of other people and the rest of creation, not to further our own ends or challenge God's sovereignty. As Christians, we must take responsibility for what mathematics we develop and how it is applied in the world around us. Our overall motivation should be service, not mastery and control. In the chapters that follow, we will develop these ideas in further detail and present a positive alternative to both the modern and the postmodern outlooks on mathematics.