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Logical "Paradox": A Response to Fallacies, Flaws, and Flimflam #36

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Abstract
Response to "Fallacies, Flaws, and Flimflam" by Ed Barbeau in *The College Mathematics Journal*, Vol. 23, No. 3 (May, 1992), pp. 203-206. Fallacies, Flaws, and Flimflam is a column in *The College Mathematics Journal* edited by Ed Barbeau of the Department of Mathematics at the University of Toronto.

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FFF #36. A Logical "Paradox"

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Using a truth table, it is shown that (*p* -> *q*) V (*q* -> *p*) is a tautology, from which it is concluded that either an implication or its converse must be true. (See *CMJ* 22 (1991) 132.) From Calvin Jongsma of Dordt College, Sioux Center, Iowa, comes the comments:

This paradox hinges on at least two confusions. The first one blurs logical syntax and semantics. Although *p* -> *q* V *q* -> *p* is a tautology under the conventional truth value definition for ->, which means for any sentences *p* and *q* that either *p* -> *q* or *q*-> *p* is true, we may not conclude that either "*p* implies *q*" is true or "*q* implies *p*" is true. Logical implication cannot be captured by this or any other truth functional connective. Thinking that it can leads to several paradoxes of implication such as this one.

Secondly, a universal quantifier has been illegally distributed in the particular example provided to make the given statement seem more paradoxical. Since the sentence "if *n* is prime, then n is odd; or if *n* is odd, then n is prime" is always true, the universal closure "for all natural numbers *n*, if *n* is prime, then *n* is odd; or if *n* is odd, then *n* is prime" is also true. However, the distributed universal disjunction "for all natural numbers *n*, if *n* is prime, then *n* is odd; or for all natural numbers *n*, if *n* is odd, then *n* is prime" need not be true (and of course isn't). All numbers are even or odd, for instance, but it's not the case that all of them are even or all of them are odd. Thus, it is invalid to distribute the universal quantifier over a disjunction. Such errors result from failing to be clear about the position of quantifiers in informal mathematical statements.

In subsequent correspondence, he pointed to the problem of "blurring the distinction between the semantic notion of logical implication (not a truth functional operator on propositions, but a logical relation between them) and the conventional truth function syntactic operator ' ->'." Further, the conditional connective ought not be to read as "implies." "The Deduction Theorem of propositional logic allows you to get away with translating ' ->' as 'implies' (or 'proves') in certain contexts but not here. The confusion between ' ->' and 'implies' is somewhat analogous to confusing the operation of division with the binary relation of 'divides,' something students tend to do when they first meet the notation *a* | *b* ."