Faculty Work: Comprehensive List

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Give Saxon the Ax!

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GIVE SAXON THE AX!

I. Introduction and Disclaimer

  A. Take a Head–Count of People Familiar with Saxon Program; How Many Appreciate Saxon? (Carrying Coals to Newcastle?? Not Earlier!)

  B. What Prompts My Evaluation of Saxon's High School Mathematics Program?
    1. Initial acquaintance with texts in preparing for a Math Day talk on 'Completing the Square' in March, 1987
    2. Attended Saxon talk at NCTM meeting in November, 1987
    3. More and more calculus students now entering with Saxon background
    4. My daughter and son are presently studying Saxon math
    5. Am bothered enough by what I see to want to share my observations with other math educators
    6. Will focus on the high school program; not the elementary school textbooks

  C. How Will I Evaluate the Books?
    1. Fair–minded evaluation, I think: both good and bad features; one much shorter than the other, though
    2. Tend toward negative assessment; parts may seem blasphemous to the Faithful
    3. Haven't had time to compare Saxon with other high school texts
    4. Not high school teacher, so my evaluation may be unwittingly skewed

II. Saxon the Man; Saxon the Educator

  A. Professional Background, Educational Experience
    1. Engineer(?), combat pilot in Air Force
    2. Air Force Academy instructor: taught ???
    3. Taught algebra at junior college in Oklahoma after retiring from AF

  B. Educational Context: Crisis in Mathematics Education
    1. Personal: students couldn't remember what they were taught; decided to teach them everything all the time
    2. National: gathering crisis in mathematics education: *David Report*; *Everybody Counts*; *Renewing U.S. Mathematics*; *Reshaping School Mathematics*; NCTM *Curriculum and Evaluation Standards for School Mathematics*

  C. Saxon Program: A New Old Vision for Mathematics Education
    1. Developed unorthodox program of algebra
    2. Fair success, especially in schools that traditionally do poorly
      a. Self–made millionaire due to sale of texts; over 1 million sold; mortgaged home to begin publication
      b. Openly challenges others to match his results; offers school systems sets of books to test them out

3. Love – hate relation with mathematics educators: prompts extreme reactions
   a. Saxon is obnoxious maverick; not part of professional mathematical education establishment
   b. Speaks at NCTM conferences; advertises in NCTM publications
   c. Abrasive personality, unorthodox program has gained Saxon strong enemies among upper echelon mathematics educators; books banned in 22 states
   d. Some rank and file teachers, including many with minimal mathematical training, are devotees of the program; have seen it work for them; his program fills a need to address failure in mathematics
   e. Saxon probably appeals to some teachers because it matches the way they already teach: they've given up on explanation, on mathematics as a rational enterprise, because most kids don't care to know why something is true, only what recipe to apply

III. The Saxon Method of Learning Mathematics
   A. Saxon's Overall Goals: Laudable
      1. Increased mastery of material
      2. Improved scores on (standardized) college entrance tests
      3. Retention of students for more advanced mathematics, science courses
      4. Make more career options available to average students

   B. Program Objectives
      1. Automation of skills: lower, mid–level, advanced
      2. Complete mastery of mathematical techniques

   C. Educational Procedures
      1. Learn through continual repetition: 'Do everything all the time'
         a. Nearly constant practice, repetition of problem–types
         b. Borders on mindless drill at times
      2. Gradual incremental development
         a. Do manageable, small chunks of new material
         b. *Can't* do too much new, if continue all problem types from earlier lessons

IV. Merits of Saxon's Program
   A. Achievements of Program (Statistics Promoted by Saxon): Can You Argue with Success?

   B. Confident but Realistic View of the Student
      1. Expectations: ordinary students can do it with work
      2. Students need to adopt a regimented, Protestant work ethic: rigid discipline required!

C. Pedagogical and Curricular Strengths
    1. Reinforcement, familiarity, mastery of skills through repeated practice
        a. Recognizes need for disciplined work, for active participation on students' part
        b. Recognizes need to keep ideas fresh through review if want to keep them functional
    2. Slow–paced development: new material kept to a minimum
    3. Emphasis on science–related topics
        a. Scientific topics from physics (uniform motion, force vectors); from chemistry (compounds; gas laws); biology (exponential growth)
        b. Unit analysis; unit conversion factors; estimation and approximation
        c. Use of subscripted variables, as in scientific work
    4. Some specific strengths
        a. Recognition of pitfalls (not all, though): e.g.: construction of equations for word problems via stages, with wrong ones first
        b. Stress on diagrams for problem–solving
        c. Honesty about contrived problems purely for practice; but not all types called contrived need be

V. Serious Drawbacks to Saxon's Program: Critique and Illustrations

  A. Unorthodox and Unnatural Curricular Organization
    1. Backward–looking curriculum: the standard critique
        a. Saxon does not promote his own content, only a method; old–fashioned material, may be addressed in later texts, versions: does some probability, statistics, etc. in III, IV
        b. Saxon consciously bucks modern trend toward letting machines do the mechanical work in number crunching, symbol manipulation
    2. Atomized morsels do not make a meal: $\delta \to 0$ in some lessons
        a. Some lessons are very small; probably too small
        b. No real synthesis takes place in Saxon; no big problems putting many things together
    3. You can't see the forest for the trees: lessons, lessons, and still more lessons
        a. Lessons follow in sequence without being grouped into chapters
        b. Students don't get any overview from the structure of the text; lessons seem to follow in rather random fashion
    4. "Why is this done now?" Principle: the next topic may *never* be done next!!
        a. To give adequate time to practice little bits before continuing, you cannot proceed to the next related topic until a week or two later
        b. Intervening lessons are thus used as filler or parallel courses; rarely connected (an exception: binomial expansion treated just before power rule)

5. Juxtaposed hodge–podge is not the same as integration
    a. A repetitive, incremental approach makes genuine integration very difficult, if not impossible;   **Cf. II LL 56 ff; 99 ff**
    b. Coherence, synthesis can't be promoted by atomizing everything into tiny pieces
    c. Saxon's claims about integration are pure rhetoric; unsubstantiated
    d. Case studies
       i. Completing the square
         Opportunity to show interplay of geometry and algebra with visually completing the square (history of mathematics: solving equations with geometrical demonstration)
         Saxon would not really be interested in such explanations, reinforcement; not part of stream–lined mechanization
       ii. Complex numbers
         Opportunity to show interplay of geometry and algebra in interpreting complex numbers as two–dimensional quantities; Saxon waits a year! Then "botches it" by using i in two different senses: as complex number, unit vector: graphs $-2 + 4i$ as $-2i + 4j$
      iii. Limits and Differentiation
         Saxon discusses limits of sums and products (without their proofs) in IV, L 79 — after doing derivatives and antiderivatives of sums and products earlier; never mentions the connection between them

B. Questionable Pedagogical Strategies
  1. Rote drill generates a sense of bored familiarity and maybe contempt, not understanding and insight
    a. Agree: certain amount of drill needed for consolidation of skills; but not to generate conceptual understanding
    b. Saxon answers critics of drill that every sport and artistic performance requires a great deal of practice, drill of skills; yet there is more to athletics and art and math than technique and reflex actions
    c. Better students are surely turned off by the repetition of everything all the time; becomes keep–busy work for them; very pedestrian program in spots
    d. Saxon teaches some skills simply to perform well on standardized tests; some topics denigrated, but done because often appear on standardized tests; we may deserve this response with emphasis on such tests   **IV passim**
  2. Everything is a mechanical skill: no room for cultural context, conceptual insight, or genuine applications
    a. Cultural context; concepts; skills; applications and interconnections: all are part of mathematics, but not according to Saxon
    b. Everything is a skill to be automated and mastered by drill / repetition

3. Problem–solving automated: templates for your every need (Engineering gone berserk)

   a. Saxon thinks problem–solving is applying ready–made templates to problems similar to ones already seen; he makes fun of texts that include problems for exploration before students have the machinery to work them (advertisement in NCTM)

   b. Template Y to fill for problem X (some rather idiosyncratic)

   c. Models to follow in working problems: even same problems!

   d. Comforting mathematics: no real thinking needed, just robotic reaction to stimuli; don't confuse me with what something means

   e. Analytical problem solving ability not developed; probably hindered in some instances (where no prior example seen)

   f. Varieties of templates needed to be remembered may be more work than working it without any template!

   g. Case studies

      i. Ratio problems
      Mechanical cross multiplication procedure **I, 109–10**

      ii. Uniform motion problems
      Many varieties spawned; types begin to overwhelm student; **I, II**
      Just recall that $D = R \cdot T$ and analyze problem's data!

C. Instructional Deficiencies

1. Robots with blinders don't explore or extend; they follow rules

   a. No exploration, discovery prior to learning a concept or extensions of concept beyond rules given

   b. Rules to be followed as per example

   c. Students not well–prepared by Saxon to extend concepts to new situations or develop their own techniques

2. "Because I said so!" Lack of motivation, explanation, conceptual understanding

   a. Authoritarian approach to mathematics; not an exercise of rational faculties; 'WHY?' is a big unanswered question; makes the avoidance of a difficult matter (conceptual explanation) into a virtue

   b. Little attempt to explain why certain big topics (e.g. algebra, complex numbers, trigonometry) are useful, not even in general terms, until long after topic is en route: II L13 introduces algebraic symbolism; L 31 finally shows some limited value to doing algebra

   c. Individual topics rarely introduced by motivating examples or initial exploration

   d. Little attempt to explain conceptual meaning of individual topics; understanding comes by repeating procedures learned by example, not through explanation of meaning; do now, learn later: **Quote: I, vi; II, v; II, 14**

   e. Exact opposite of present trend to relegate certain mechanical procedures to computers and calculators and concentrate on conceptual understanding

   f. Dulls intellectual curiosity instead of encouraging it; H. Jacobs vs. J. Saxon

   g. Creates problems for later teachers (us!) who insist on explaining things; students don't want it, aren't able to conceptualize well

h. Case studies

    i. Equation of a straight line
Graph of first degree equation plopped on student as straight line, to be accepted as fact; ditto for slope constant (even worse case) **I, 153; 236**

    ii. Similar figures
Bald statement defining similar figures as those whose angles are identical and whose sides are proportional; no explanation why such figures can exist, satisfying this conjunction; not a definition, but a definition and an important theorem combined

    iii. Pascal's triangle
Shows how Pascal's triangle is useful for finding coefficients of a binomial expansion;
Claims nothing to be explained or understood (**Quote: III, 280**); certainly is! can note that binomial coefficients are found using factorials; can show the relation among these that is apparent in the triangle

    iv. Complex numbers
Numbers are ideas; extend $\sqrt{a}\sqrt{a} = a$ to negative numbers to get imaginary numbers; but can't extend $\sqrt{a}\sqrt{b} = \sqrt{ab}$: why not? The rule is just different for different signed numbers, that's why! **II, 179–180**
Does mathematics of complex numbers mechanically by following certain algebraic rules, without explanation
Missed opportunities to explain confusion mathematicians had over complex numbers; how they arose with solving the cubic; how they were finally interpreted geometrically as two–dimensional quantities and so legitimized

    v. Derivatives: presents derivatives of exponential, logarithmic and trigonometric functions without any motivation or justification; ditto for the Fundamental Theorem of Calculus

3. The emasculation of robust mathematics: lack of justification, proof

    a. Not everything needs rigorous proof, of course

    b. Yet concept of proof is important to advanced mathematics; should be done properly

    c. Students should start to see mathematics as a deductive, rational enterprise, even though there is more to mathematics than proof

    d. Saxon's curriculum fails to deliver on this aspect, even in calculus text

    e. Case studies

        i. Trigonometry
Proving trig identities: confusion generated by Saxon
Seems not to understand what's going on (**quote: III, 276**)
Promotes false proof procedures (**III, 387**); transforms each side into other forms, proceeding from equation to equation; can be made legitimate, but now students should wonder why they can't use all rules for equations had before!

        ii. Deductive geometry
Prototype of axiomatic, deductive reasoning for high school students;
Saxon geometry rather loose; early in the book results are used to prove

certain results and work problems; later in the book they are proved from other results; has circularity occurred? Do students know they should avoid circularity in their proofs? Not from Saxon.

Axiomatic approach in Saxon geometry is simply awful, using similar triangles as basis of triangle geometry; axiom is far from elementary: it's both a definition and a theorem

Congruence made to depend upon similarity; makes simple topic complex; constructions done late in text (but stated without proof)

Usurps congruency criteria / acronyms for similarity (SAS for similarity!)

Similarity depends upon theory of parallels and theory of proportion; rejected as basis for geometry by Euclid due to complexities involved, if done right

    iii. Calculus text includes some proofs, and even makes a show of introducing some logic at the outset (poorly done; not continued or emphasized later); but less is done here than in standard texts on college level.

D. Some General Criticisms

  1. Sponsors an enfeebled mathematical sensibility

    a. Students do not receive good intuition, overview from Saxon

    b. Students do not learn how to become problem–solvers from Saxon

    c. Students do not receive good sense of proof from Saxon

  2. No sense of mathematical history: lack of cultural context, appreciation

    a. Only history in Saxon that I saw is wrong: Egyptians did not have Pythagorean theorem, so far as we know; Babylonians and Chinese did

    b. No use of history for determining when in general to do what, when in the curriculum

  3. Incompetent presentation of some topics: a few examples

    a. Poor proofs of trigonometric identities: mentioned above

    b. Limits presented without adequate understanding of what they are, what they are for; claims functions with point discontinuities are pathological and useful only for illustration purposes; seems not to realize that difference quotients, which lead to derivatives, are precisely of this sort

    c. Integration techniques: claims that the chief technique of antidifferentiation is "guessing" and checking; failing that, try substitution (more guessing advocated), integration by parts, partial fractions

    d. His approach to a given topic often does not hold up under close scrutiny; he lacks genuine mathematical sensibility. There are wrong or imprecise statements, irrelevant materials, invalid arguments (non–sequiturs), poor organization and pedagogy. In short, Saxon's curriculum is a travesty perpetrated upon the mathematical community. How did educators allow it to happen, and how can we continue to do nothing about it?

VI. Prospectus

A. Where Should We Go from Here?

  1. Genuine need addressed by Saxon's program; has struck a responsive chord in the American public: we need to (continue to) address poor mathematical performance / ability of U.S. high school students

2. Need for cooperation on all levels to evaluate curriculum, methods of instruction, to participate in curriculum construction

3. Need to encourage further professionalization of mathematics teachers on pre–college level; start in math education programs

B. Goals of School Mathematics Education Reconsidered

   1. NCTM *Standards*

   2. What are *our* goals?

      a. Students: who are we teaching? Why?

      b. Mathematical content: what dictates inclusion of certain topics? how should they be organized? Should Saxon's sequence of topics make us rethink how we do things?

      c. Mathematical methodology: what is appropriate for different age levels? What role should repetition and continual review play in learning mathematical skills and ideas? (Picked up by some new series, I believe)

      d. Cultural context of mathematics: how has mathematics developed, and what might that tell us about content organization, methodology, and teaching strategies?

      e. Religious–philosophical context of mathematics: how do we as Christians participate in mathematics?