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Neglected Standard: History of Mathematics in the Service of Mathematics Education

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Neglected Standard: History of Mathematics in the Service of Mathematics Education

Abstract

An integrated approach that more intrinsically connects the historical development of mathematics with its content enables students to learn how an idea or method emerged while simultaneously exposing (i) the dynamic nature of mathematics along with (ii) its connections to other fields, and (iii) its cultural embeddedness. Examples will be given from a textbook currently in development.

Keywords

history of mathematics, mathematics education

Disciplines

Christianity | Mathematics

Comments

Presentation from the 17th Biennial Conference of the Association of Christians in the Mathematical Sciences held on the campus of Wheaton College in Wheaton, Illinois, May 27-30, 2009.

History of Mathematics in the Service of School Mathematics Education

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May 28, 2009

OUTLINE

- ✦ Textbook Project Description
- ✦ HoM for Math Educators
- ✦ Greek Ratio & Proportion
 - ∝ Numerical Theory of R&P
 - ∝ Educational Ramifications
 - ∝ Geometric Theory of R&P
 - ∝ Use of Geometric R&P
 - ∝ More Educational Ramifications
- ✦ Concluding Remarks

✦ Textbook Project Description in Brief

- ▶ Course for Prospective Middle School Mathematics Teachers
 - Curricular Identity Crisis: Changing Educational Scene
 - Refocused, Using History of Mathematics
- ▶ Need for Class Materials
 - Homemade Exercises
 - Extensive Outlines
 - Textbook Sections: in process

✦ History of Mathematics for Mathematics Educators

- ▶ Cultural Benefits
 - Cultural Matrix of Mathematics
 - Multi-Cultural Contributions
 - Embedding Mathematics in an Integrated Social Studies Unit
- ▶ Views of the Nature of Mathematics
- ▶ Origin and Development of Ideas and Techniques
 - Ontogeny Recapitulates Phylogeny?
 - Cognitive Development, with Cultural Differences
 - HoM Alerts Educators to Difficulties and Distinctions
 - HoM Alerts Educators to Developmental Prerequisites
- ▶ Pedagogical Implications

✦ Greek Ratio and Proportion

- ▶ Meaning of Terms
- ▶ Origins and Early Development
 - Pre-Greek Use of Ratio and Proportion
 - Slope of inclines
 - Work rates
 - Strength of bread/beer
 - Greek Theory of Ratio and Proportion
 - Cultural Context: Rational Explanation/Foundation
 - Numerical Theory of Pythagoreans [Euclid VII]
 - Geometric Theory of Eudoxus [Euclid V]

► Numerical Theory of Ratio and Proportion

- Ratio
 - Relation in respect of size between two quantities [V, Def.3]
 - **Not** a fraction; and **certainly not** a number
- Proportion
 - Sameness of Ratios? (Euclid V)
 - Proportional when same part, parts, or multiple [VII, Def.20]
- Proportion Explicated
 - *Part*: when it measures the whole; submultiple [VII. Def.3]
 - *Parts*: smaller, but not a part [VII, Def.4]
 - *Same Part*: same multiplicative relation, same unit fraction
Ex: 3 is the same part of 12 that 5 is of 20: the fourth-part
 - *Same Parts*: same number of parts of same part-type
Ex: 8 is the same parts of 12 that 10 is of 15: 2 third-parts
 - Proportional numbers
Ex: $3 : 12 :: 5 : 20$; $8 : 12 :: 10 : 15$
- Modernized Criterion: proportional iff same fully reduced form

► Educational Ramifications of Numerical Theory of R & P

• Ratio

- Meaning of ratio as a multiplicative relationship is central
- Connection between ratio and fractions/numbers: takes time?
- Comparison to MSM Ed textbooks: ordered pair definition
 - * Far too abstract for learning what a ratio is
 - * Wrong; not abstract enough for a formal treatment

• Proportion

- Meaning of proportion as same fully reduced ratio is intuitive
- Euclid's criterion stays within realm of ratios/fractions
- Comparison to MSM Ed textbooks: cross multiplication criterion [VII.19]
 - * Less fundamental?
 - * Easy to apply

► Geometric Theory of Ratio and Proportion

• Ratio

- Relation in respect of size between two quantities [V, Def.3]
- Magnitudes have a ratio iff they are capable, when multiplied, of exceeding one another [V, Def.4]
 - * Homogeneity requirement
 - * Not infinitesimal/infinite with respect to one another
 - * Can be incommensurable with one another (key!)

• Proportion

- *Proportional*: sameness of ratios [V, Def.6]
- *Same Ratio*: any common multiple of the first and third alike exceed, equal, or fall short of any common multiple of the second and fourth respectively [V, Def.5]
- Heterogeneity permitted in proportions

- Use of Geometric Ratio and Proportion
 - Mathematics
 - * Area
 - * Volume
 - * Similar figures; indirect measurement
 - Kinematics
 - * Uniform motion/constant speed: $d_1 : d_2 :: t_1 : t_2$
 - Mechanics
 - * Law of Lever: $d_1 : d_2 :: w_2 : w_1$
 - * Centroids of regions
 - Beginnings of quantitative science/mathematization

► Educational Ramifications of Geometric Theory of R & P

- Ratio

- Homogeneous ratios: fit multiplicative meaning of ratio best
arise first? simplest to understand?
- Heterogeneous ratios: rates arise late (early modern era)
easier for today's students? still conceptually complex?
- Comparison to MSM Ed textbooks: little attention paid to the
difference between homogeneous and heterogeneous ratios

- Proportion

- Heterogeneous magnitudes can be related by abstract
magnitude-neutral ratios between homogeneous quantities
- Cross multiplication criterion leads to compound magnitudes
- Ratio and proportion: the backbone of mathematical science

✦ Concluding Remarks on Project

- ▶ Value of History of Mathematics for (Middle School) Mathematics Teachers
 - Overview and Perspective
 - Background and Breadth/Enrichment
 - Comprehension of Topics' Meaning and Significance
- ▶ Value of History of Mathematics for the (Middle School) Classroom
 - Overview and Perspective
 - Background and Breadth/Enrichment
 - Comprehension of Topics' Meaning and Significance
- ▶ Questions or Suggestions?