What is Number?

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What is Number?

by Willis Alberda
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Most mathematicians and philosophers consider number to be a primitive concept. That is, the concept of number cannot be reduced to or identified with any concept that is more primitive. The idea of number cannot be further defined with words whose most immediate intuitive meaning is more obvious or basic than the word number. This, coupled with the fact that number is an abstract entity which cannot be observed, felt, or handled in a concrete way, gives rise to diverse theories of the structure of number.

In most mathematics discussions this difficulty is pointed out rather vaguely or entirely overlooked. Attempts to clarify the number structure fall into two broad categories. Some define natural numbers as the process of counting, a process independent of time and space and an imme-


c) Measuring number: if a magnitude is measured, it is exhausted or tried to be exhausted by means of copies of a unit, like a vessel is emptied with a scoop.

d) Reckoning number: this is the algorithmic aspect. The number is operationally comprehended, by rules according to which the user plays with it. 2

Both approaches give inadequate answers to the question: What is number? If number is only a creation of the human mind, then is there no structure to this concept other than that democratically agreed upon by mathematicians through the ages? If number is defined to be an ordinal number, cardinal number, rational

"Before undertaking this analysis, there are three basic givens which set the perspective for this discussion. First, on the basis of a faith commitment and without further argument, we will assume that number is a created entity .... Second, we believe that the number aspect is ... like all of creation subject to law. Third, number is, ... a distinct aspect of the creation in at least one fundamental respect. Equivalently, number is an irreducible aspect of the creation."

The purpose of this paper is to give a succinct, brief analysis of the essence of number. This analysis does not replace or contradict the theories discussed above, but is an attempt to get at a biblically based prior concept out of which the theory of numbers can be developed.

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first steps of which are as arduous for children as learning the names of colours and letters, until they suddenly grasp the whole unlimitedly continuing sequence. A conceptual seizure that has no analogue in learning the names of colours and letters.

b) Numerosity number: perhaps the numerosity number is genetically earlier than the counting number. Animals recognize small numerosities though they certainly cannot count.

d) Reckoning number: this is the algorithmic aspect. The number is operationally comprehended, by rules according to which the user plays with it.

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the universe. Number was considered the basic counting unit, the basic geometric unit, and the basic physical unit. (We now identify the latter two as point and atom or element respectively.) Through further theoretic and scientific developments, it became apparent that the entities number, point, and atom were not identical.

Arithmetic for the Greeks was basically a discrete, finite theory, and problems arose when points on a line were matched on a one-to-one basis with the points on a longer line (Zeno's Paradox) or when diagonals of squares were measured with the same units as those used to measure the side of the same square.

The problem they overlooked was that number is a distinct entity not reducible to other equally irreducible entities. The following may illustrate this point. The symbol 1 standing alone conveys an immediate concept to each observer in western culture. If, however, this symbol is associated with each of the following—(1, —), (1, Δ), (1, □)—the most immediate concept is different in each of the three symbols. Even though these three entities all have a measure of 1, the line segment, the triangle, and the square each adds a new and distinct dimension to the concept which the symbol 1, alone, cannot convey. By way of contradiction, one might argue that if the symbol 1 were sufficient to represent all three
All of these examples point out the fact that we live in, are part of, and respond to a creation where distinctions are possible and are consciously or unconsciously made to avoid confusion. The reasons for these distinctions cannot always be proven logically or derived, but are often an intuitive response to a matter of fact. The history of natural science and mathematics gives many examples of where not making such distinctions has resulted in confusion and loss of direction in the development of the science.

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used to identify the units of time, space, velocity, and force, to name just a few, we see evidence of the irreducibility of one creational aspect to that of another. This the Greeks did not recognize.

In arithmetic, that is, in operations with numerals, number symbols, we need no units description. One object is represented by the symbol 1. But in geometry, the science of space, we commonly assign very specific types of units such as 1 inch, 1 square foot, or 1 cubic yard, to name only a few. When studying physical phenomena—that is, when mass is involved—we customarily use a different set of units, such as 1 pound, 1 gram, or similar weight or mass units. The units change again for the study of the motion of a point. Here we use 1 inch per second or 1 mile per hour and so on; and the units change once more as we discuss the motion of a mass-particle, when the units used are 1 foot-pound per second or 1 dyne, newton, or horsepower.

Only man has the ability to abstract and symbolize number, but experiments with animals show that the numerical sense is not unique to man. It is a created entity evidenced in many ways in the creation. Some plants show a unique number relation, called the Fibonacci Sequence in the number of leaves and branches the plant develops and their relative positions on the plant. Robins usually lay four eggs. Experimenter have removed one egg from the nest and the mother robin lays another, repeatedly. Even though the conclusions are controversial and other plausible explanations might be offered, there is accumulated a mass of evidence supporting the belief that certain birds, certain mammals, and certain arthropods perhaps have a number sense. The following anecdote may more directly illustrate the point.

There is a touching and authentic story about a bird that seemed to possess a number sense. A squire
try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth. From this statement, which summarizes most parents' experiences when teaching a child how to count, it is quite obvious that the raw material for the development of the number concept is inherently present before the child is ready to handle the concept abstractly. Many types of experiments have been conducted which show that a child can carry out the basic counting process, setting a one-to-one correspondence between objects, before that child can verbalize, or through rational action, illustrate the process.

We always deal with numbers abstractly, and attempts to make the idea concrete fail because there is no specific thing which can be identified uniquely with the concept number. It is for this reason that number is so very difficult to describe. This is not the case for the other basic mathematical concepts. A point in space, for example, can be illustrated as the place where two walls and the ceiling of a room meet. This concretizes the concept of point. But there is no such representation of number.

This problem is bothersome to mathematicians in general and can be summarized by the following statement by Kuyk:

The question whether it is possible to make some kind of ontology the basis of modern mathematics is left open by most people working in the mathematical fields. Fearing to introduce into mathematics arguments of a metaphysical nature, the philosophically minded mathematician will avoid as much as possible reference to mathematical existence independent of human thought. In general it can be said that under the impact of the pragmatist attitude, for the philosopher of mathematics the workability of mathematical systems rather than their interpretability has become a central point of

To develop further background and insight into the essence of number, we look at how children learn numbers. Piaget says, "It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults

in Scotland became annoyed by a raucous crow that had made its nest in the watchtower of his estate, and he determined to shoot the bird. Repeatedly he tried to enter the tower to kill the bird, but each time at the man's approach the crow would leave its nest and take up a watchful position in a distant tree. When the wearied squire would leave the tower, the bird would return to its nest. Not wishing to be outsmarted by a bird, the squire resorted to a ruse. He secured the assistance of a neighbor one day. The two men entered the tower; one man came out and went away, and the other remained within. But the crow was not deceived; it stayed in the distant tree until the man within the tower came out. The experiment now became a contest, and the next day three men entered the tower, two came out and went away, and the third waited within. But the crow was not fooled; it remained in the distant tree until the man within the tower came out. The next day the experiment was repeated with four men, but still without success. Finally five men entered the tower, four came out and went away, and the fifth remained inside. At this point the crow seemed to have lost count and, unable to distinguish between four and five, it returned to its nest in the tower.
view. Reflections of an epistemological nature as well as reflections regarding for example mathematical truth are not readily undertaken by mathematicians of the pragmatistic type. To break with the usual ontology of number, whether intuitionistic, logistic, or formalistic, it will be necessary to recognize first that the idea of natural number cannot be defined. The concept of the natural number "one" cannot be reduced to or identified with any concept that is more primitive. Moreover, we will recognize the existence of number independent of human thought. Number is a concept integral to the whole of creation and man structures this and other mathematical concepts appropriately only when he does so according to the laws placed in the creation by the Creator.

Since number is a primitive concept, our discussion of what is number cannot be further refined by definition, but will have to take the form of ontology by creation laws or norms. The concept of number can only be recognized as existing, and hence as mathematical truth, because it was created so, and it can be described only in terms of a set of boundaries or laws beyond which the entity ceases to be number.

Within this context, I believe, the idea of number derives its meaning out of the created fact of identity. When the Creator formed the cosmos and each rock, plant, and creature in it, he was concerned with the details of His creation. Implicit in the "And God saw that it was good," is the identification of each of the products of His handiwork and the knowledge of what it was. Explicit in the "All according to their kind" is the fact of differentiation, distinction, and distinguishability. These two phrases carry with them the very direct message of identity. Whatever was created, was done so according to the way in which God intended it, and could never be otherwise. With this attribute present in the creation, it is possible to make distinctions, selections, and choices.

Fundamentally, the aspect of the number "one" is abstracted, through identification and selection, from an integral object in the horizon of immediate experience, in distinction from all other aspects of that object. Very simply, it is the process of saying "this one" and not "that one." Piaget expands on this as follows:

In analysing the beginnings of quantification, we find ourselves confronted with the problem of correspondence. To compare two quantities is indeed either to compare their dimensions, or to make a one-to-one correspondence between the elements. As a result of the work of Cantor, the second of these processes has been seen to be fundamental to the construction of the integer, since it provides the simplest and most direct measurement of the equivalence of two sets.

Identification of a single object carries with it the number one. Repeated identifications give rise to correspondences and, hence, to the construction of the integers. The reason for discussing the number concept in terms of the number "one" is that most discussions of this type begin with two or more and then attempt to define the process of counting and also numbers in terms of classification sets or equivalent sets. This brings one too quickly to the process of counting, without clearly illus-
In the area of pedagogical methods in arithmetic, we note that set theory per se cannot and will not be the basis for number concepts. Number concepts and the arithmetic operations of numbers can be clarified and illustrated by using sets, but not defined or derived from set theory. This may be one of the problems associated with the so-called "new math" programs, where sets are made the basis for the number concept, rather than a method of illustrating this concept.

Even though the number concept is very difficult to define and discuss and has eluded mathematicians and philosophers over the centuries, we should not hesitate to clarify this concept. Number is a part of God's creation given to us to use to His glory. If we make number abstract and elusive, when it is in fact a very naive and intuitive sense given to man to use, we are remiss in overlooking its simple beauty. If we keep this in mind, however, we may well increase the effectiveness of our service to God and our fellowman.

FOOTNOTES