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Irrational Numbers and Reality

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"There are no such things as irrational numbers; all numbers are rational. All numbers make sense."

This chance remark of a colleague over the coffee cup set me to thinking. Of course there are no "unreasonable" numbers. When a mathematician speaks of irrational numbers, he means those numbers which cannot be written as a ratio of two integers, as contrasted with a rational number (such as 3/4), which <u>can</u> be written that way. But where did this idea of "unreasonable" numbers come from?

The ancient Greeks made repeated attempts to explain reality in its most simple terms. One example of this was Empedocles' classification of all elements into four: air, water, earth, and fire. The Greeks also noticed the close relation of reality to mathematics, and one group of philosophers, the Pythagoreans, believed all of reality could be reduced to number. Hence, to gain control of number implied control of all reality. This school spent considerable time and effort looking for patterns in the numbers, developing various number systems and studying the relation between reality and number.

They had discovered the natural numbers, (1, 2, 3---), and by considering all ratios of natural numbers had developed the rational number system, i.e. all fractions.

It seemed to them for a time that they had developed the ultimate number system. Using it, they were able to find answers to all the problems posed.

Eventually, however, one of them succeeded in demonstrating that there were problems with no solution in the rational numbers, i.e., he showed that it was not possible to express $\sqrt{2}$ as the ratio of two natural numbers. Not only this, but using the familiar Pythagorean theorem, it follows that no matter what unit of length is chosen as a standard, there will always be lengths which cannot be expressed as units or ratios of units. For with any standard unit one can construct a right triangle whose legs are both that unit's length. This triangle will have an hypotenuse of length $\sqrt{2}$, and hence is not expressible in units or a ratio of units. Such numbers were called irrationals and such lengths incommeasurables.

As a result, the Greek mathematical world was shattered. Legend tells us that this disturbing fact was uncovered in Syracuse in southern Italy, and as its discoverer was sailing back to Greece, he was thrown overboard, so that his discovery might die with him.[1] This would indicate that these people certainly took their mathematics seriously!

From our vantage point today we tend to smile a bit at their naiveté. But let us be careful not to laugh too soon. Although the Greeks did not acknowledge a sovereign Creator, they clearly saw the orderliness of His creation, and the extremely close relationship between that creation and the systems of mathematics. Many of us today are not aware of this, as we ought to be.

Historically, mathematics was derived from the real world. Men noticed numerical relationships, spatial relationships, etc., in nature, and abstracted the mathematical aspect. From the 19th century on, however, developments in the real world have followed rather than led developments in mathematics.

"From time to time certain mathematical breakthroughs were discarded because they were thought out of step with reality."[2] But during this century it was discovered that these breakthroughs did fit certain aspects of reality. Riemann's non-Euclidean geometry, for example, fit certain aspects of interstellar motions, whereas Euclid's geometry did not. Mathematics became more abstract and developed at a rapid rate, so that, for example, when Einstein was toying with his idea of general relativity, he found that the mathematics necessary to substantiate his theory, i.e., Tensor Analysis, had already been developed (1887-1896) by Gregorio Ricci-Curbastro.[3] Thus, he who had control of mathematics really had control of reality. The technological revolution of the 20th century followed from this mathematical revolution in the 19th.

Bourbaki also recognizes this when he states, "From the axiomatic point of of view, mathematics appears thus as a storehouse of abstract forms—the mathematical structures; and it so happens—without our knowing why—that certain aspects of empirical reality fit themselves into these forms, as if through a kind of preadaptation."[4] Bourbaki had in mind, for example, group theory, a study of the most basic properties found in every number system, whose structures have found applications in modern Quantum Mechanics.

Here is one more instance of "having eyes, they see not." Mathematically mankind has progressed a long way from the ancient Greeks. Yet, while the orderliness of creation is clearly displayed, man fails to acknowledge his Creator.

What is our response? Are we properly awed when we see the majesty of God displayed in this aspect of our lives? May our response be a genuine and sincere "Soli Deo Gloria"!

Footnotes

1. Carl B. Boyer, A History of Mathematics, John Wiley and Sons, New York, 1968, p. 79.

2. Francis J. Mueller, "What's Happened to Our Mathematics?", Providence Sunday Journal, Rhode Island, March 18, 1962.

3. Morris Kline, Mathematical Thought from Ancient to Modern Times, Oxford University Press, New York, 1972, p. 1131.

4. N. Bourbaki, "The Architecture of Mathematics," American Mathematical Monthly, Vol. 57, (1950), pp. 221-232.