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# Existence in Mathematics

Willis J. Alberda
*Dordt College*

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by Willis Alberda
Professor of Mathematics

*Willis J. Alberda received his Ph.D. in Math from Montana State University. He has been at Dordt College since 1964 where he is Professor of Mathematics and Chairman of the Math Department.*

Contemplation of the existence of mathematical entities generates, for very apparent reasons, a cycle of arguments dealing with, 1) the nature of mathematical truth, and 2) meaning in mathematics; and there follows the obviously related question: Which of these two problems should be solved first? The problem of the existence of mathematical entities, the subject of this discourse, is integral to any attempt to elucidate the basic mathematical structures. This problem plagued the early Greek mathematicians, and, even today, is fundamental to the domain of speculation and research on the foundations of mathematics. When we try to place ourselves in the position of the philosophers who first explored this problem, we must realize that it sprang from the apparent or obvious discrepancy between the truths of mathematics and the entities to which these truths refer. The problem is that the truths of mathematics appear to belong to those areas of human knowledge to which we ascribe the highest degree of certainty; but we search vainly in the world of human experience for entities which have properties described by these truths. For example, we know that any two points determine a straight line, but nowhere in the world of human experience or the mathematical laboratory do we find points or straight lines in the proper sense of the word.

The existence of mathematical entities is both a simple and a profound issue, as observed by W.V. Quine:

> A curious thing about the ontological problem is its simplicity. It can be put in three anglo-saxon monosyllables: "What is there?" It can be answered, moreover, in a word—"everything"—and everyone will accept this answer as true. However, this is merely to say that there is what there is. There remains room for disagreement over cases; and so the issue has stayed alive down the centuries.[1]

A review of the literature dealing with the history of the problem of existence of mathematical entities reveals a persistent question: What is the fulcrum or Archimedean point from which all existence can be derived? In mathematics, possibly because of its abstractness, the mind has usually been considered to be that starting point. But that assumption has been challenged for some time. Robert J. Baum (1973) suggests that

> The question of the role of the mind in the acquisition of mathematical knowledge is one of the central topics of debate among present-day philosophers of mathematics. The basic disagreement on this matter is over two issues—whether the objects of mathematical knowledge exist independently of the mind prior to their being known or are in some way created by the mind, and whether mathematical theories are discovered or created by the mind. The terminological distinction between "discovery" and "creation" is of value for identifying certain prejudices not only of philosophers but also of some mathematicians whose writings appear to be of a purely mathematical nature.[2]

The relevance of this statement is reflected in the confusion of perspectives in contemporary schools of mathematics. The logicists used the basic principles of logic to develop the mathematical structures and used only those methods of proof which have a purely logical character. Mathematicians such as Dedekind, Frege and others used this approach to the foundations of mathematics. John Stuart Mill and B. Erdmann defended the approach of mathematical empiricism and logical psychologism as a method of answering foundational problems. The intuitionists, such as Kant and especially E. J. Brouwer, claimed "basal intuition" as the cornerstone for the structures of mathematics. Later Hilbert introduced mathematical formalism, which claimed to solve the existence problem in mathematics by making mathematics a purely formal or symbolic system.[3]

In all of these schools, the concept of the infinite is a crucial issue. The countability of the infinite and the concept of "actual infinity" were problems which had their beginnings in the discourse of Greek philosophers and mathematicians. Aristotle, Zeno, and Democritus all had early formulations of the problem. Many mathematicians struggled with the problem, notably Galileo in the late sixteenth century, Bolzano in the early nineteenth century, and particularly Cantor who attempted to solve the problem using sets. Cantorism, popular in the nineteenth century, built the foundations of mathematics on set theory. The set concept thus became the building block for other mathematical entities.

Intuitionism, as anticipated by Kant and in recent times propounded by mathematicians such as L. Kronecker, H. Poincaré, and as systemically developed by Brouwer and his school, maintains that mathematics is independent of logic and that mathematical entities and theories should sustain intuitionistic criticism. In this view, intuition is meant to be that *a priori* power of the mind which knows it can conceive of repeating the same act indefinitely, when the act is once possible.[4] This means that certain inferences can be made without the support of logic or Cantorian theorems. Brouwer's approach demonstrates the nonanalytical character of mathematics, but yet allows for the inferential transition from a synthetic presupposition of mathematical existence to the formal consistency of a set of axioms.

The formalists' approach to mathematics deprives mathematics of the meaning sought by the other approaches and makes mathematics a system of consistent sets of symbols and axioms. The emphasis here is language analysis, the language being the accepted mathematical symbols. Existence of the mathematical symbols in such a system is of only incidental concern. Consistent use of the symbols is imperative in this system, and no challenge of their existence is invited.

It becomes quite evident, however, from reading W. Kuyk, that the question of existence is always answered from the perspective of a fulcrum or Archimedean point or, as one might say, of a faith commitment which becomes the source of all existence.

Kuyk writes:

> Basically, the answers given to this question went in two different directions: a) the Cantorist-realist-formalist attitude, which maintains that one could go as far as one likes, on the condition that no contradiction results in the system; and b) the remaining attitudes, including the idealist-intuitionistic ones and the logicistic-realistic ones which delimit the language, and let set theory depend on cosmological considerations.[5]

Although the two different directions described by Kuyk do not do justice to the complexity of each of these theories or to the theory of the foundations of mathematics implied by them, the point is that attempts to answer the question of existence of mathematical entities have never gone beyond a man-made theory of systems and methods. Basically the methods used to answer existence questions in mathematics, and for that matter in most sciences, have been and still are objective constructionism and subjective idealism. They are man-centered, not Creator-centered. The scientist and mathematician persistently begin and end with the creation itself.

The constructionist sees existence in the concrete entities of mathematics which can and, to his mind, do exist independent of human intervention of analysis. These *a priori* objects then become the building blocks of mathematical theories. The idealists rely heavily on the rational capability of the human mind and its ability to formulate meaningful relations. Outside of human comprehension or intuition, no real mathematical structures exist, because for the man-centered scientist the essence of mathematical entities is human-rational, lingual-symbolic expression.

Taking the approach of the realist-constructionist we find the basic mathematical entities dependent upon intuitively clear concepts. The approach of the idealist-logicist-formalist, on the other hand, claims to find these basic entities in an abstract, axiomatic analysis. The problem in the first position is that of demonstrating these entities in an unequivocal, concrete way. In the second approach we face a complete reliance on the logical, rationalistic, analytic foundation. Both approaches are reductionistic because, on the one hand, mathematics deals with entities which for the most part are impossible to demonstrate concretely, and, on the other hand, mathematics is distinct from logic and linguistics.

Any attempt at resolution of the existence problem must be divested of the problems inherent in the existing theories. If we recognize that any theory of mathematical existence proceeds directly from a given perspective, philosophy, or faith commitment, it is incumbent upon all Christians to establish their perspective, philosophy, or faith commitment in accord with the will of God. Thus we seek guidance from the Scriptures to establish the context from which we attempt to answer all questions of life, and particularly the questions of the academy and scholarship.

Specific to the question of existence, the Scriptures clearly show that 1) God is, 2) God made all that there is, 3) each created entity, whether seen or unseen, has been made to glorify the Creator, 4) as each part of creation responds in obedience to the Creator, it brings glory to the Creator, 5) man as the crown of creation is mandated to fully develop creation to the glory of the Creator, 6) man and with him all of creation fell into sin because of disobedience, and 7) through the saving work of Christ, man, by faith, is again restored to a loving relation with the Creator and can again respond in obedience to the laws of the Creator, although hindered by sin.

These confessional statements do not define a philosophy of mathematics nor do they answer the questions of existence in mathematics. They do, however, give a perspective which can be used to develop a philosophy of science or mathematics which will eliminate some of the theory-related problems described earlier. The philosophy which follows is not the only one that could be developed, but is one which seriously attempts to work out the religious commitment of the Christian.

If all there is was created and is upheld by the Creator, then all entities of the cosmos, including the mathematical, exist and will continue to exist as an integral part of the creation No entity exists independent of all other entities. Each entity is related to the Maker and hence finds meaning in that relation and the relations it sustains to other created entities. These relations are governed, as is all of creation, by the will of the Creator, and this will we may call "law." In obedience to this law entities within the creation become meaningful. Entities exist, then, only as they are part of a meaning relation. It is man's role in this cosmos to come to a clearer understanding of these meaning relations and to elucidate these relations in an opening of the creation to its intended purpose. The composite of these endeavors is the work of man in the creation, the carrying out of the cultural mandate.

Basically, my answer to the existence question rests upon the view that all created things, visible and invisible, are subject to law. Existence then becomes a matter of correlating entity and law, making meaning possible. If we recognize that all of reality has both a law side and an entity-subject-to-law side, and that a correlation of these two brings meaning and implies existence, then some resolution of the problems of existence inherent in the views described earlier is possible. Rationalism (idealismlogicism-formalism), which reduces mathamatics to a study of logic or formal statements, emphasizes law at the expense of subject. Intuitionism (realism - empiricism - constructionism), which reduces mathematics to a study of objects or abstractions, concentrates on the entity or subject while slighting the law side. Without a doubt, both elements in proper balance bring about a further unfolding of the realm of mathematics. In some instances abstraction may precede concrete formalization and in other cases the reverse may be true, but historically, all mathematics which has come to be known as mathematics has had an appropriate balance of both elements. This is not accidental, but an integral fact of creation; it is an illustration of the wisdom of the Creator.

Doing mathematics is a matter of uncovering mathematical entities and the laws which hold for these entities. Another way of saying this might be that the unfolding process in mathematics is the correlation of laws and entities subject to law. The laws govern the relations between entities; the entities are real entities which in turn share their reality with those entities to which they relate under law. Stafleu (1974) went so far as to say that there are no entities without laws; every entity is constituted by some law, and related to other entities according to laws. Also, there are no laws without possible or actual entities.[6] If we accept this position, the existence question in mathematics becomes a matter of formulating new law statements, identifying entities which obey given law statements, or refining both law statement and abstract entity for fuller clarity.

In this context, the number two, for example, is the numeric subject entity which provides the meaning relation between the abstract concept of "twoness" and the experiential sets or collections of two objects. This entity cannot have full meaning, however, until it is related to other numeric entities by a relation such as "more than," a law to which it is subject. Similarly, the geometric subject entity, line, conveys the concept of the space where two walls meet, but is subject to the relation of being determined by two points. Obviously, these are examples of mathematical entities whose existence few will argue, but careful analysis will show that the foundational arguments which plague the mathematical world today, even though the arguments are much more sophisticated, lend themselves to this method of resolution. The fruitfulness of this approach for examining the foundations of mathematics has been given some substance in early chapters of the work of Stafleu (1974). Although not explicitly stated, the third chapter of the work of Kuyk (1977) may contain a theory of the foundations of mathematics implied by this view of mathematical existence. If the theories of the foundations of mathematics are to be unified, I think it will require research with the orientation I have attempted to describe here.

## Footnotes

1. Quine, W. V.: "On What There Is," in *Philosophy of Mathematics*, ed. by P. Benecerraf and H. Putnam, Prentice-Hall Inc., Englewood Cliffs, New Jersey; 1964; p. 183.
2. Baum, Robert J.: *Philosophy and Mathematics - From Plato to the Present.* Freeman, Cooper & Co., San Francisco; 1973; p. 9.
3. Beth, Evert W.: *The Foundations of Mathematics.* North-Holland Publishing Company, Amsterdam; 1968.
4. Poincaré, H. : *Science and Hypothesis.* (1902) Dover, New York; 1952; p. 13.
5. Kuyk, W.: *Complimentarity in Mathematics.* D. Reidel Publishing Co., Dordrecht-Holland; 1977; p. 13.
6. Stafleu, M.D.: *A Systematic Analysis of the Foundations of Physics.* An unpublished manuscript; 1974.