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Abstract

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Is the $Y(2175)$ a strangeonium hybrid meson?

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I. INTRODUCTION

The initial state radiation (ISR) process in e^+e^- annihilation is a very useful technique to search for vector states (i.e., $J^{PC} = 1^{--}$) in B-factories. In 2006, the *BABAR* Collaboration studied the cross sections for the ISR processes $e^+e^- \to K^+K^-\pi^+\pi^-$ and $e^+e^- \to K^+K^-\pi^0\pi^0$ up to 4.5 GeV, aiming to confirm the existence of the $Y(4260)$ in the $\phi\pi\pi$ channels. Instead of observing the $Y(4260)$, however, they found a new resonance structure in the $\phi(1020)f_0(980)$ channel, which was named the $Y(2175)$ [1]. (It is also known as the $\phi(2170)$ [2]). This resonance was later confirmed by *BABAR* [3–5], BES [6], and Belle [7] and recently by BESIII [8,9]. Its mass and decay width are $M = (2188 \pm 10)$ MeV and $\Gamma = (83 \pm 12)$ MeV and its quantum numbers are $I^G J^{PC} = 0^-1^{--}$ [2].

To date, the nature of the $Y(2175)$ is still unknown. Based on strange quarkonium mass predictions using a relativized potential model, only the 3^3S_1 and 2^3D_1 $s\bar{s}$ states are expected to have masses close to that of the $Y(2175)$ [10]. However, both interpretations are disfavored as the corresponding resonance width predictions are significantly larger than the width of the $Y(2175)$. The width of the 3^3S_1 $s\bar{s}$ state was predicted to be 378 MeV using the $^3P_0$ decay model [11] whereas the width of the 2^3D_1 $s\bar{s}$ state was predicted to be 167 MeV in the $^3P_0$ model and 212 MeV in the flux tube breaking model [12]. Another possible interpretation of the $Y(2175)$ is that of a strangeonium hybrid meson (i.e., $\bar{s}gs$). Masses of vector strangeonium hybrid mesons have been computed using several methodologies including the flux tube model [13–16], lattice QCD [17], and QCD Laplace sum rules (LSRs) [18]. The flux tube model calculation of [16] found a vector strangeonium hybrid mass of 2.1–2.2 GeV. The lattice QCD analysis of [17] found a vector strangeonium hybrid mass between 2.4 GeV and 2.5 GeV while the LSRs calculation of [18] found a heavier mass of (2.9 ± 0.3) GeV. Yet another possible interpretation of the $Y(2175)$ is that of a $ss\bar{s}\bar{s}$ tetraquark. In [19], the masses of vector $ss\bar{s}\bar{s}$ tetraquarks were investigated. Two states were predicted with respective masses (2.34 ± 0.17) GeV and (2.41 ± 0.25) GeV. Other LSRs analyses of $ss\bar{s}\bar{s}$ tetraquarks can be found in [20,21]. Furthermore, the $Y(2175)$ has also been proposed as a molecular state of $\Lambda\bar{\Lambda}$ [22,23]. In [22], a chromomagnetic interaction Hamiltonian was used to predict a hexaquark of mass 2.184 GeV that is strongly coupled to the $\Lambda\bar{\Lambda}$ channel. In [23], a one-boson-exchange potential model was used to predict a $\Lambda\bar{\Lambda}$ mass between 2.149 GeV and 2.181 GeV. Also, the $Y(2175)$ has been interpreted as a dynamically generated resonance of $\phi f_0(980)$ [24–27].

Decay modes and rates will be crucial to determining the nature of the $Y(2175)$. In [11], it was predicted using the $^3P_0$ model that the most important decay modes of the 3^3S_1 $s\bar{s}$ meson would be K^*K^*, $KK^*(1410)$, and $KK_1(1270)$ whereas the KK mode would be very weak. In [12], it was

predicted using the $^3P_0$ model that the most important decay modes of the 2^3D_1 $s\bar{s}$ meson would be $KK(1460)$, $KK^*(1410)$, $KK_1(1270)$, and K^*K^*. While not dominant, the KK decay mode was predicted to have a partial width of about 0.06. In [28], it was predicted using flux tube and constituent gluon models that the most important decay modes of a vector strangeonium hybrid would be $KK_1(1400)$, $KK_1(1270)$, $KK^*(1410)$, and $KK_2(1430)$, each containing a S-wave meson plus a P-wave meson, due to the $S+P$ selection rule [28–30]. Also, it was noted that the $\phi f_0(980)$ mode could be significant. Of particular interest are the KK, K^*K^* and $KK(1460)$ modes which are predicted to be zero for a strangeonium hybrid interpretation (the usual rule that suppresses or even forbids hybrid decays to pairs of S-wave mesons [29,31,32]). For the $ss\bar{s}\bar{s}$ tetraquark interpretation, it has been suggested that the $\eta\phi$ channel should be one of the dominant decay modes due to the large phase space in the fall-apart mechanism [28]. However, in [33], it was argued that the $\eta\phi$ decay mode would be greatly suppressed and that the $\phi f_0(980)$, $h_1\eta$, and $h_1\eta'$ modes would be most important. For the $\Lambda\bar{\Lambda}$ interpretation of the $Y(2175)$, the KK decay mode was predicted to dominate [34]. At present, the data concerning decay modes and rates of the $Y(2175)$ is incomplete, making it difficult to draw any definitive conclusions [2].

As they are both observed in ISR processes, the $Y(4260)$ and $Y(2175)$ states have the same quantum numbers, and are often considered as analogous states in the hidden-charm and hidden-strange sectors, respectively [1,35,36]. Perhaps determining the nature of one will shed light on the other. Since the $Y(4260)$ has been identified as a good candidate for charmonium hybrid $\bar{c}gc$ [37–39] or hidden-charm tetraquark state $qc\bar{q}\bar{c}$ [40], the $Y(2175)$ meson may also be interpreted as a hybrid or tetraquark candidate.

In this work, we use QCD Gaussian sum rules (GSRs) methods to study the strangeonium hybrid possibility for $Y(2175)$. In contrast to previous analyses of strangeonium hybrids using LSRs [18], the use of GSRs enables an exploration of the possibility of multiple states with hybrid components, allowing us to examine the scenario of a hybrid component of the $Y(2175)$. We find little evidence in support of the $Y(2175)$ having a significant strangeonium hybrid component.

II. THE CORRELATOR AND GAUSSIAN SUM RULES

We investigate vector strangeonium hybrids through the correlator

$$\Pi(q^2)=\frac{i}{D-1}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)\int d^Dx e^{iq\cdot x}\langle\Omega|\tau j_\mu(x)j_\nu(0)|\Omega\rangle \tag{1}$$

where D is spacetime dimension and where the current j_μ is given by

$$j_\mu=\frac{g_s}{2}\bar{s}\gamma^\rho\gamma_5\lambda^a\tilde{G}^a_{\mu\rho}s. \tag{2}$$

In (2), s is a strange quark field and $\tilde{G}^a_{\mu\rho}$ is the dual gluon field strength tensor,

$$\tilde{G}^a_{\mu\rho}=\frac{1}{2}\epsilon_{\mu\rho\omega\zeta}G^a_{\omega\zeta}, \tag{3}$$

defined in terms of the Levi-Civita symbol, $\epsilon_{\mu\rho\omega\zeta}$.

Between [18] and [41], the quantity $\Pi(q^2)$ from (1) has been computed to leading-order (LO) in $\alpha_s=\frac{g_s^2}{4\pi}$ within the operator product expansion. In [18], the perturbative and dimension-four (i.e., 4d) quark and gluon condensate contributions were calculated. In [41], the 5d mixed, 6d quark, and 6d gluon condensate contributions were calculated as well as $\mathcal{O}(m_s^2)$ corrections to perturbation theory where m_s is the strange quark mass. Denoting the result as $\Pi^{\mathrm{QCD}}(q^2)$ to emphasize that it is a QCD calculation, we have

$$\Pi^{\mathrm{QCD}}(q^2)=\left(\frac{\alpha_s}{\pi}\left(-\frac{q^6}{240\pi^2}+\frac{5m_s^2q^4}{48\pi^2}-\frac{4q^2}{9}\langle m_s\bar{s}s\rangle\right)+\frac{q^2}{36\pi}\langle\alpha G^2\rangle+\frac{19\alpha_s m_s}{72\pi}\langle g\bar{s}\sigma Gs\rangle\right)\log\left(\frac{-q^2}{\mu^2}\right) \tag{4}$$

where

$$\langle m_s\bar{s}s\rangle=\langle m_s\bar{s}^\alpha_i s^\alpha_i\rangle \tag{5}$$

$$\langle\alpha G^2\rangle=\langle\alpha_s G^a_{\mu\nu}G^a_{\mu\nu}\rangle \tag{6}$$

$$\langle g\bar{s}\sigma Gs\rangle=\langle g_s\bar{s}^\alpha_i\sigma^{\mu\nu}_{ij}\lambda^a_{\alpha\beta}G^a_{\mu\nu}s^\beta_j\rangle \tag{7}$$

are respectively the 4d strange quark condensate, the 4d gluon condensate, and the 5d mixed strange quark condensate. In (5)–(7), subscripts on strange quarks are Dirac indices, superscripts are color indices, and $\sigma^{\mu\nu}=\frac{i}{4}[\gamma^\mu,\gamma^\nu]$. In computing (4), divergent integrals were handled through dimensional regularization in $D=4+2\epsilon$ dimensions at $\overline{\mathrm{MS}}$-renormalization scale μ. A dimensionally regularized γ_5 satisfying $\{\gamma_5,\gamma^\mu\}=0$ and $\gamma_5^2=1$ was used following the prescription of [42]. Also, TARCER [43], a *Mathematica* implementation of the recurrence relations of [44,45], was employed to reduce the set of needed integral results to a small, well-known collection. An irrelevant polynomial in q^2 has been omitted from (4) as it ultimately does not contribute to the GSRs used in this article (see below). Included in this omitted polynomial are the 6d quark and gluon condensate contributions, both of which are constant for this channel as discussed in [41].

The quantity $\Pi(q^2)$ in (1) is related to its imaginary part, i.e., the hadronic spectral function, through a dispersion relation

$$\Pi(Q^2) = \frac{Q^8}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{Im}\Pi(t)}{t^4(t+Q^2)}\mathrm{d}t + \cdots \tag{8}$$

at Euclidean momentum $Q^2 \equiv -q^2 > 0$. In (8), t_0 is a hadron production threshold and $\cdots$ represents subtraction constants, collectively a third degree polynomial in Q^2. On the left-hand side of (8), we identify Π with Π^{QCD} of (4). On the right-hand side, we partition the hadronic spectral function using a resonance-plus-continuum decomposition,

$$\frac{1}{\pi}\mathrm{Im}\Pi(t) = \rho^{\mathrm{had}}(t) + \theta(t-s_0)\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{QCD}}(t), \tag{9}$$

where $\rho^{\mathrm{had}}(t)$ represents the resonance contribution to $\mathrm{Im}\Pi(t)$ and $\theta(t-s_0)$ is a Heaviside step function shifted to the continuum threshold parameter s_0.

In (8), to eliminate subtraction constants as well as the aforementioned polynomials omitted from (4) and to enhance the resonance contribution relative to the continuum contribution to the integral on the right-hand side, some transform is typically applied leading to some corresponding variant of QCD sum rules. Laplace sum rules, for example, are a common choice (e.g., see [46–49]). Here, we instead choose to work with (lowest-weight) GSRs defined as [50]

$$G(\hat{s},\tau) = \sqrt{\frac{\tau}{\pi}}\lim_{\substack{N,\Delta^2\to\infty\\ \tau=\Delta^2/(4N)}}\frac{(-\Delta^2)^N}{\Gamma(N)}\left(\frac{d}{d\Delta^2}\right)^N\left\{\frac{\Pi(-\hat{s}-i\Delta)-\Pi(-\hat{s}+i\Delta)}{i\Delta}\right\}. \tag{10}$$

Discussions of how to evaluate definition (10) for a correlator such as (4) can be found in [50–52]. Substituting (9) into (8) and applying (10), we find

$$G^{\mathrm{QCD}}(\hat{s},\tau) \equiv \frac{1}{\sqrt{4\pi\tau}}\int_0^{\infty} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{QCD}}(t)\mathrm{d}t \tag{11}$$

$$\Rightarrow G^{\mathrm{QCD}}(\hat{s},\tau) = \frac{1}{\sqrt{4\pi\tau}}\int_{t_0}^{\infty} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\rho^{\mathrm{had}}(t)\mathrm{d}t + \frac{1}{\sqrt{4\pi\tau}}\int_{s_0}^{\infty} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{QCD}}(t)\mathrm{d}t. \tag{12}$$

Subtracting the continuum contribution,

$$\frac{1}{\sqrt{4\pi\tau}}\int_{s_0}^{\infty} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{QCD}}(t)\mathrm{d}t, \tag{13}$$

from (11) and (12) leads to subtracted GSRs

$$G^{\mathrm{QCD}}(\hat{s},\tau,s_0) \equiv \frac{1}{\sqrt{4\pi\tau}}\int_0^{s_0} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{QCD}}(t)\mathrm{d}t \tag{14}$$

$$\Rightarrow G^{\mathrm{QCD}}(\hat{s},\tau,s_0) = \frac{1}{\sqrt{4\pi\tau}}\int_{t_0}^{\infty} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\rho^{\mathrm{had}}(t)\mathrm{d}t. \tag{15}$$

Finally, calculating $\mathrm{Im}\Pi^{\mathrm{QCD}}(t)$ from (4) and substituting the result into the right-hand side of (14), we find

$$G^{\mathrm{QCD}}(\hat{s},\tau,s_0) \equiv \frac{1}{\sqrt{4\pi\tau}}\int_0^{s_0} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\left(\frac{\alpha_s}{\pi}\left(-\frac{t^3}{240\pi^2}+\frac{5m_s^2t^2}{48\pi^2}-\frac{4t}{9}\langle m_s\bar{s}s\rangle\right)+\frac{t}{36\pi}\langle\alpha G^2\rangle+\frac{19\alpha_s m_s}{72\pi}\langle g\bar{s}\sigma Gs\rangle\right)\mathrm{d}t. \tag{16}$$

Note that the definite integral in (16) can be evaluated in terms of error functions. The kernel of the subtracted GSRs is a Gaussian of width $\sqrt{2\tau}$ centred at $\hat{s}$. As discussed in [41,51–53], GSRs are particularly well suited to the study of multiresonance hadron models as, by varying $\hat{s}$, excited and ground state resonances can be probed with similar sensitivity.

Renormalization-group (RG) improvement of (16) amounts to replacing α_s and m_s by running quantities at the scale $\mu^2 = \sqrt{\tau}$ (e.g., [50,54]). The one-loop, $\overline{\mathrm{MS}}$ running coupling at $n_f = 4$ active quark flavors is

$$\alpha_s(\mu) = \frac{\alpha_s(M_\tau)}{1+\frac{25}{12\pi}\alpha_s(M_\tau)\log(\frac{\mu^2}{M_\tau^2})} \tag{17}$$

where [2]

$$M_\tau = 1.77686 \pm 0.00012\ \text{GeV} \tag{18}$$

$$\alpha_s(M_\tau) = 0.325 \pm 0.015. \tag{19}$$

Since the previous analysis of strangeonium hybrid mesons using QCD sum rules [18], the condensate parameters and quark masses are now known more precisely. In addition to the inclusion of higher-dimensional condensates terms in (4), we update the values and uncertainties in the QCD parameters used in [18]. The running strange quark mass is

$$m_s(\mu) = m_s(2\ \text{GeV})\left(\frac{\alpha_s(\mu)}{\alpha_s(2\ \text{GeV})}\right)^{\frac{12}{25}} \tag{20}$$

where [2]

$$m_s(2\ \text{GeV}) = 96^{+8}_{-4}\ \text{MeV}. \tag{21}$$

The value of the RG-invariant 4d strange quark condensate is known from the partially conserved axial current,

$$\langle m_s\bar{s}s\rangle = -\frac{1}{2}f_K^2 m_K^2, \tag{22}$$

where [2,55]

$$m_K = (493.677 \pm 0.016)\ \text{MeV} \tag{23}$$

$$f_K = (110.0 \pm 4.2)\ \text{MeV} \tag{24}$$

For the 4d gluon condensate, we use the value from [56],

$$\langle \alpha G^2\rangle = (0.075 \pm 0.020)\ \text{GeV}. \tag{25}$$

For the 5d mixed strange quark condensate, we use the estimate from [57,58],

$$\frac{m_s\langle g\bar{s}\sigma G s\rangle}{\langle m_s\bar{s}s\rangle} \equiv M_0^2 = (0.8 \pm 0.1)\ \text{GeV}^2. \tag{26}$$

Integrating (15) with respect to $\hat{s}$ gives

$$\int_{-\infty}^{\infty} G^{\text{QCD}}(\hat{s},\tau,s_0)\mathrm{d}\hat{s} = \int_{t_0}^{\infty}\rho^{\text{had}}(t)\mathrm{d}t. \tag{27}$$

The quantity on the left-hand side (LHS) of (27) is the lowest-weight finite energy sum rule (FESR), and, as shown in [50], the spectral function decomposition (9) only reproduces the QCD prediction at high energy scales if s_0 is constrained by (27). To isolate the information in the GSRs that is independent of the FESR constraint (27), we define normalized GSRs (NGSRs) [51],

$$N^{\text{QCD}}(\hat{s},\tau,s_0) \equiv \frac{G^{\text{QCD}}(\hat{s},\tau,s_0)}{\int_{-\infty}^{\infty} G^{\text{QCD}}(\hat{s},\tau,s_0)\mathrm{d}\hat{s}} \tag{28}$$

which, from (15) and (27), implies that

$$N^{\text{QCD}}(\hat{s},\tau,s_0) = \frac{\frac{1}{\sqrt{4\pi\tau}}\int_{t_0}^{\infty} e^{-\frac{(\hat{s}-t)^2}{4\tau}}\rho^{\text{had}}(t)\mathrm{d}t}{\int_{t_0}^{\infty}\rho^{\text{had}}(t)\mathrm{d}t}. \tag{29}$$

III. ANALYSIS METHODOLOGY AND RESULTS

Previous work using LSRs used a single-narrow resonance model and resulted in a strangeonium hybrid mass prediction significantly heavier than the $Y(2175)$ [18]. Compared with that analysis, we include 5d and 6d condensate terms in the operator product expansion and use updated QCD parameter values. Also, as outlined above, Gaussian sum rules have the ability to probe multiple states in the spectral function. We can therefore update and extend the analysis of Ref. [18] and test the hypothesis of a $Y(2175)$ hybrid component by using a double-narrow resonance model for the hadronic spectral function

$$\rho^{\text{had}}(t) = f_1^2\delta(t-m_1^2) + f_2^2\delta(t-m_2^2). \tag{30}$$

This double narrow-resonance model in (15) provides the hadronic contribution, i.e., the right-hand side, to the NGSRs (29),

$$N^{\text{had}}(\hat{s},\tau) = \frac{1}{\sqrt{4\pi\tau}}\left(re^{-\frac{(\hat{s}-m_1^2)^2}{4\tau}} + (1-r)e^{-\frac{(\hat{s}-m_2^2)^2}{4\tau}}\right), \tag{31}$$

where the normalized couplings are defined as

$$r = \frac{f_1^2}{f_1^2+f_2^2}, \qquad 1-r = \frac{f_2^2}{f_1^2+f_2^2}, \qquad 0 \le r \le 1. \tag{32}$$

We fix one of our modeled resonances (m_1) using the experimental value given in Refs. [2,59],

$$m_1 = m_{Y(2175)} = 2.188\ \text{GeV}, \tag{33}$$

and the additional resonance (m_2) provides the necessary degrees of freedom in the model for the possibility that the $Y(2175)$ decouples (i.e., that m_1 has normalized coupling $r \approx 0$).

We choose the width of the Gaussian kernel to be $\tau = 10\ \text{GeV}^4$, in line with our previous GSRs analysis of light hybrids [41]. Since this resolution is much larger than the experimental width of the $Y(2175)$ (i.e., $\sqrt{\tau} \gg m_1\Gamma$), the narrow width model is an excellent approximation for the $Y(2175)$. For the undetermined resonance m_2, we assume that it is similarly narrow compared to the Gaussian kernel resolution; this assumption is revisited in the results of our analysis

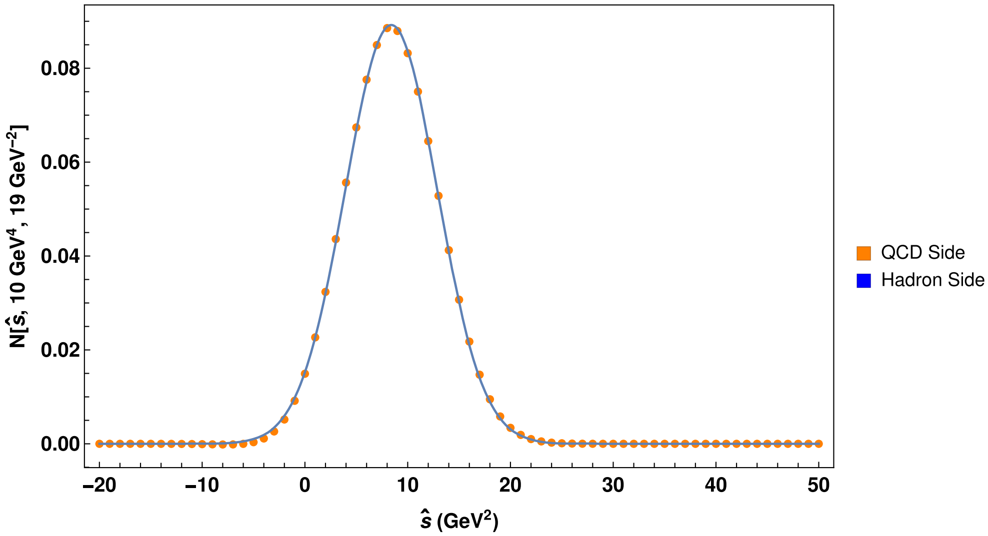

FIG. 1. Double-narrow resonance model $N^{\mathrm{had}}(\hat{s},\tau)$ (solid blue curve) and compared to QCD prediction $N^{\mathrm{QCD}}(\hat{s},\tau,s_0)$ (orange points) for $\tau = 10\ \mathrm{GeV}^4$. Central values of the QCD condensates and the corresponding fitted parameters have been used.

presented below. To determine the remaining unknown quantities $\{m_2, r, s_0\}$ in our model we seek the best fit of the $\hat{s}$ dependence of the QCD prediction and hadronic model by minimizing the χ^2,

$$\chi^2(r, m_2, s_0) = \sum_{\hat{s}_{\min}}^{\hat{s}_{\max}} [N^{\mathrm{had}}(\hat{s},\tau) - N^{\mathrm{QCD}}(\hat{s},\tau s_0)]^2, \quad (34)$$

where we use 161 equally spaced $\hat{s}$ points with $\hat{s}_{\min} = -10\ \mathrm{GeV}^4$ and $\hat{s}_{\max} = 30\ \mathrm{GeV}^4$. This region safely encloses the resonances resulting from our analysis as outlined below. Note that the minimization is constrained by the physical condition $0 \le r \le 1$ in (32). The resulting prediction of the resonance parameters and continuum onset is

$$s_0^{\mathrm{opt}} = 9.7 \pm 1.0\ \mathrm{GeV}^2 \quad (35)$$

$$m_2 = m_{\mathrm{fit}} = 2.90 \pm 0.16\ \mathrm{GeV} \quad (36)$$

$$r \le 0.033. \quad (37)$$

The uncertainties in (35)–(37) are obtained by varying the values of the QCD input parameters, and calculating the deviation from the central values in quadrature. Errors are dominated by the variation in $\langle \alpha G^2 \rangle$. An upper bound on r is provided because of the $r \ge 0$ constraint. Figure 1 shows that the fit between the QCD prediction and hadronic model is excellent; there is no evidence of any deviations that would suggest a need to refine the model (e.g., inclusion of a numerically large width $\sqrt{\tau} \sim m_2\Gamma$ for m_2). Figure 1 also shows that the fitted region $-10\ \mathrm{GeV}^2 < \hat{s} < 30\ \mathrm{GeV}^2$ encloses the regions where the NGSRs are numerically significant. As a further validation of our results, we note that our mass prediction for m_2 is consistent with previous LSRs analyses [18].

The key aspect of our results (35)–(37) is the small relative resonance strength $r \le 3.3\%$ of the $Y(2175)$ compared to m_2, which seems to preclude a predominant hybrid component of the $Y(2175)$. We can obtain a more conservative bound on r by calculating the s_0 dependence of r (i.e., choosing s_0 and only fitting r and m_2) and then considering the variation of r within the region of uncertainty in s_0 from (35). The result of this analysis leads to the bound $r \le 5\%$ as shown in Fig. 2. A similar analysis for m_2 is shown in Fig. 3.

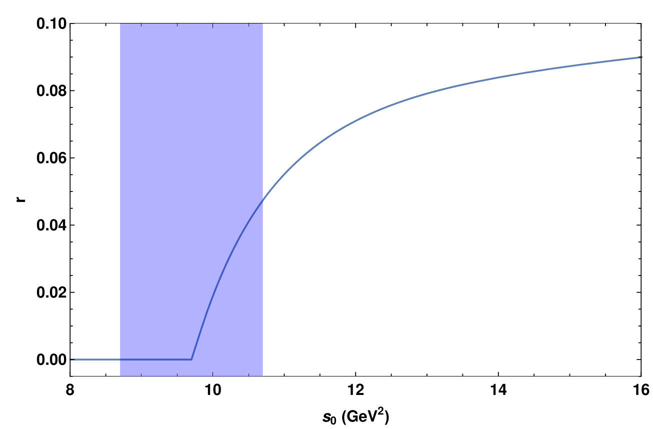

FIG. 2. Predicted coupling r to $Y(2175)$ state as a function of the continuum onset s_0. The vertical band highlights the optimized value of continuum onset s_0^{opt} with corresponding error (35). The physical constraint $r > 0$ has been imposed in the analysis.

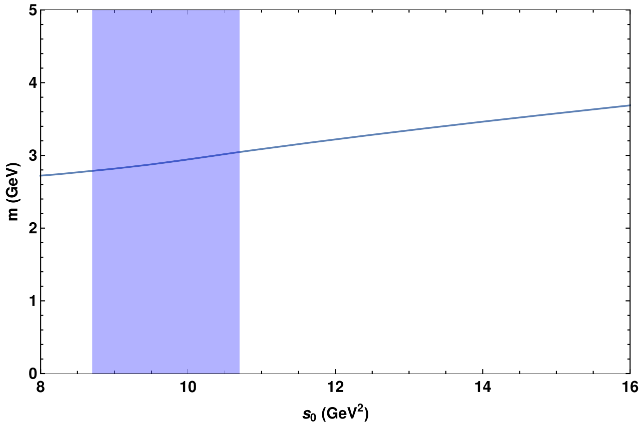

FIG. 3. Predicted vector strangeonium hybrid mass m_2 as a function of the continuum onset s_0. The vertical band highlights the optimized value of continuum onset s_0^{opt} with corresponding error (35).

IV. DISCUSSION

In summary, we have used QCD GSRs to study the strangeonium hybrid interpretation of the $Y(2175)$. Compared to a previous LSRs analysis of vector strangeonium hybrids [18], our calculation includes 5d and 6d condensate contributions, strange quark mass corrections to perturbation theory, and updated QCD parameter values. Furthermore, the advantage of the GSRs approach over the LSRs approach is its comparable sensitivity to multiple states in a hadronic spectral function. This allowed us to explore the relative coupling to the hybrid current (2) of the $Y(2175)$ and an additional unknown resonance. We found excellent agreement between the QCD prediction and hadronic model, and determined an upper bound $r \leq 5\%$ for the relative coupling strength of the $Y(2175)$. In other words, we found no evidence for a significant strangeonium hybrid component of the $Y(2175)$.

Recently, a structure of mass (2239 ± 13.3) MeV and width (139.8 ± 24.0) MeV (where we have combined statistical and systematic uncertainties) was observed in $e^+e^- \to K^+K^-$ with the BES III detector [60]. If the structure can be identified with the $Y(2175)$, then the observed KK decay mode would disfavor the 3^3S_1 strangeonium meson, strangeonium hybrid, and $ss\bar{s}\bar{s}$ tetraquark interpretations. On the other hand, if the structure cannot be identified with the $Y(2175)$, then the lack of observed KK decay mode would disfavor the 2^3D_1 strangeonium meson and $\Lambda\bar{\Lambda}$ interpretations. Clearly, further experimental and theoretical studies are needed.

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