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The Continuous, the Discrete and the Infinitesimal in Philosophy and Mathematics (Book Review)

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Abstract

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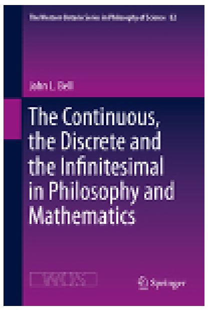
The Continuous, the Discrete and the Infinitesimal in Philosophy and Mathematics



John L. Bell

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Discreteness and continuity are core contrasting emphases in mathematics. Aristotle declared the discrete and the continuous to be the two species of quantity, the second of his ten basic philosophical categories. Prior to this, the Greeks had concluded that continuous magnitude could not be explained in terms of number/discrete quantity on account of the Pythagorean discovery of incommensurable magnitudes. In fact, they went so far as to disaggregate the two. Aristotle even stipulated that each field should be developed using only principles appropriate to its subject matter. Accordingly, Euclid's Elements included two theories of ratio and proportion—one for numbers (the Pythagorean theory of Book VII) and one for continuous magnitudes (the Eudoxan theory of Book V).

Intuitively, one readily recognizes the difference between continuity and discreteness. Continuous magnitudes are all-of-a-piece, forming an unbroken and cohesive unity, while discrete collections contain a plurality of separate members. Continuous magnitudes can be infinitely subdivided into ever smaller continua, while dividing a discrete quantity eventually yields indivisible units. The process of successive division of continuous quantity thus also leads to questions about the nature and existence of infinity and infinitesimals. These interconnected themes—discreteness, continuity, infinity, indivisibles, and infinitesimals—are the focus of The Continuous, the Discrete and the Infinitesimal in Philosophy and Mathematics. Its author, John Bell, draws upon decades of research and writing on these topics, giving the reader an ambitious historical and contemporary survey of how they have been treated over time, both mathematically and philosophically. Attempting to reduce the continuous to the discrete, Bell notes, has been one of the principal enduring endeavors of mathematics.

The first part of the book (five chapters) is a mix of philosophy and mathematics, recounting the history of these matters from ancient Greek times through the first three decades of the twentieth century. Bell begins with philosophical ideas put forward by the pre-Socratics as well as Plato and Aristotle, touching as well on the mathematical ideas embodied in the so-called method of exhaustion employed by Eudoxus and Archimedes. The first chapter also looks briefly at ancient Chinese and Indian cultures before moving on to medieval Islamic and European perspectives. Chapter 2 examines the mathematical development of infinitesimal calculus in the early modern era as well as some philosophical ideas about the nature of reality in the small. The third chapter explores different approaches to the foundations of calculus in the eighteenth century (Euler, d'Alembert, Lagrange) as well as challenges and analyses by some philosophers of the period (Berkeley, Hume, Kant, Hegel).

The next two chapters present opposing approaches to continuity. Chapter 4 tells the familiar story of how Bolzano, Cauchy, Riemann, Weierstrass, Dedekind, Cantor, and Russell developed various constructs for arithmetizing analysis, treating the linear continuum ultimately as a discrete set of points/numbers, using the set of rational numbers as a basis. Chapter 5, on the other hand, explicates some divergent conceptions of the continuum held by various nineteenth and twentieth-century philosophers and mathematicians, such as Brentano, Poincaré, Brouwer, and Weyl.

Part I requires some undergraduate knowledge of calculus and analysis to follow its mathematical narrative, but it primarily presents a historical and philosophical exposition of the ideas. While based on some standard secondary sources, such as Carl Boyer's *History of the Calculus* (which is now a bit dated), it also includes a superabundance of quotations from primary sources—roughly half of the text in the first part is devoted to quotes, making it feel a bit patched together.

The character of Part II, which focuses on continuity and infinitesimals in today's mathematics, is quite different. For one thing, it ups the ante on mathematical prerequisites by an order of magnitude. Some philosophical discussion occasionally enters the exposition, but in the main the presentation summarizes technical twentieth-century mathematical ideas and trends, so one may need a serious graduate-level introduction to the relevant fields to follow the discussion.

Part II also contains five chapters. The first three move along rather quickly: Chapter 6 covers topological spaces and manifolds in eight pages; Chapter 7 treats categories and functors, sheaves and topoi in nine pages; Chapter 8 discusses the basics of nonstandard analysis in four pages. The last two chapters are more substantive. Chapter 9 explains in 16 pages how both constructive and intuitionistic mathematics structure the real line continuum, how they handle logical operators, and how they conceptualize continuity and infinitesimals. Chapter 10 explores various facets of smooth infinitesimal analysis and synthetic differential geometry in about 40 pages, building partly upon earlier ideas from the chapters on topology and category theory but also axiomatizing the theory without assuming much knowledge of that material. This chapter also draws some comparisons with the standard approach to analysis, and it shows how parts of physics can be developed based upon its ideas.

As a trailer to entice readers to pick up Bell's book, let me quote the final sentence of the book: "It is fortunate that today, through category theory and intuiutionistic [sic] logic, the means for reviving the geometric vision [i.e., seeing the continuum as a cohesive whole instead of as a set of discrete points/numbers] are at hand."

As this quote shows, Bell's book contains careless copy-editing mistakes. In fact, the book is riddled with such problems—sloppy misspellings, repeated words, missing spaces, extra spaces, font issues, bad line breaks in mathematical notation, missing symbols, unbalanced parentheses in formulas—more than a third of the book's pages have one or more errors on them. The blurb on the back cover even misspells Riemann's name. These mistakes can usually be corrected by the reader though some will no doubt be mystified by Bell's occasional use of Russell's dot notation in place of parentheses. Production issues aside, Bell's book offers a reader with the requisite mathematical background a wide-ranging synthesis and a knowledgeable survey of the topics mentioned in its title. I found its concluding treatment of less orthodox approaches to these matters particularly thought-provoking and informative.

Calvin Jongsma (Calvin.Jongsma@dordt.edu) is a Professor of Mathematics Emeritus at Dordt University in Sioux Center, Iowa. Though he has taken graduate-level courses in topology, nonstandard analysis, and category theory, his main areas of interest are history and philosophy of mathematics, especially as related to logic and foundational developments.

Tags: Philosophy of Mathematics
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