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Projective Geometry Hidden Inside: Can You Spot It?

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Abstract

In this talk Dr. Clark shared about a Math Teachers' Circle session he recently ran centered around the children's game Spot it! This game has some very interesting mathematics behind it and naturally begs to be explored with inquiry. He described the way he led teachers to ask questions about the game, the way the teachers then explored the topic, and the mathematics behind it all.

Keywords

projective geometry, games, problem solving, axioms

Disciplines

Geometry and Topology

Comments

Presented at the 2016 MAA Mathfest held August 3-6, 2016, in Columbus, Ohio. Talk was based on the paper *Analyzing Unique-Matching Games Using Elementary Mathematics*, co-authored with Cal Jongsma.

Projective Geometry Hidden Inside:

Can You Spot It?

Thomas J. Clark

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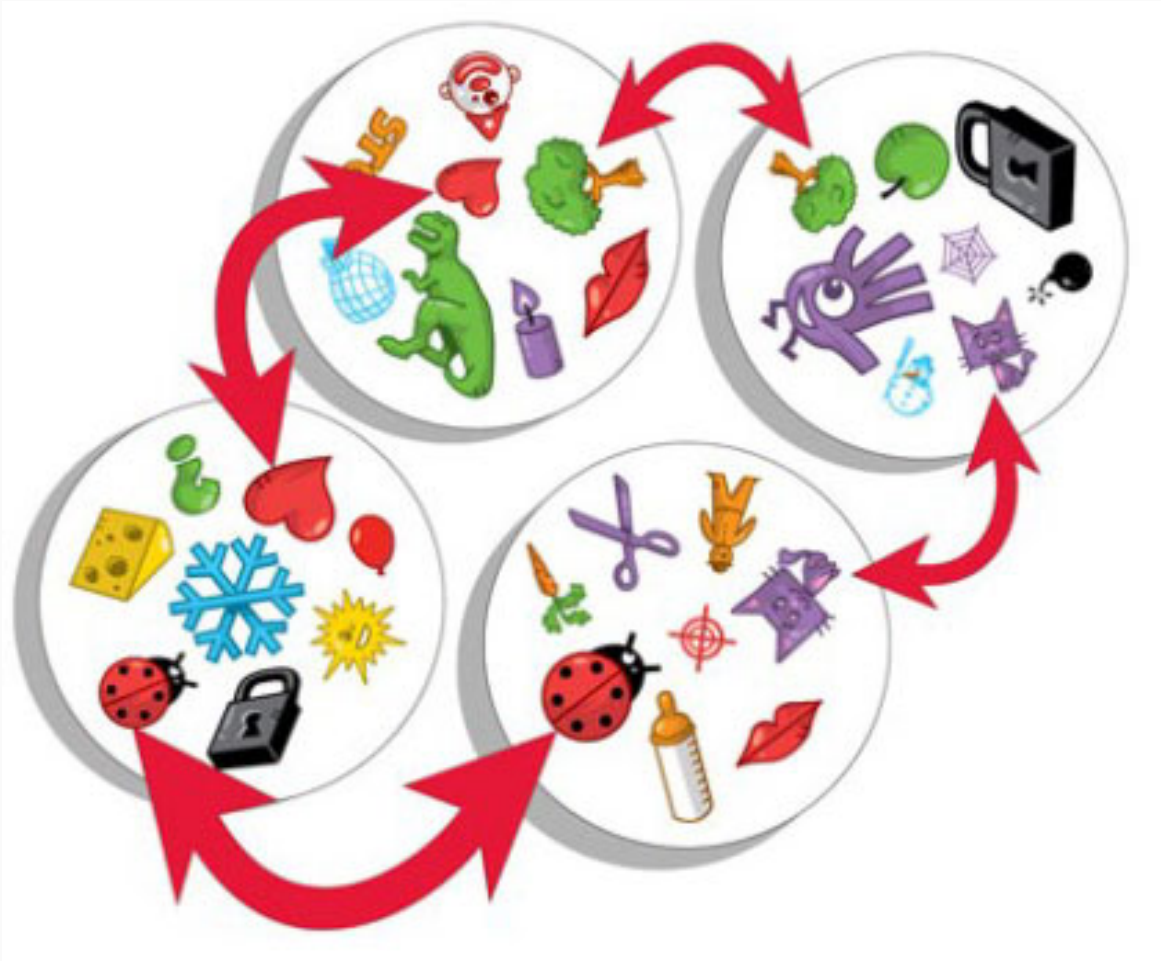
Dordt College

an introduction to spot it!

What is Spot It!^{TM}?

Basic Idea

- Every card has exactly one match with every other card.
- No “filler” symbols.



My Spot It!^{TM} Story

Curiosity

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Question

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- How many possible "decks" can one make?

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- How many times does each symbol appear? Is it the same for each symbol?
- How many possible "decks" can one make?
- How hard is it to make a Spot It!^{TM} deck of a particular size/type?

possible mathematical directions

Build Your Own Game

Problem Solving Theme: Special Cases / Simplify Problem / Patterns

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- Look for numerical patterns in that case and attempt to generalize and/or look for explanations of those patterns e.g., $s + (s-1)^2 = s^2 - s + 1$ cards given s symbols per card.

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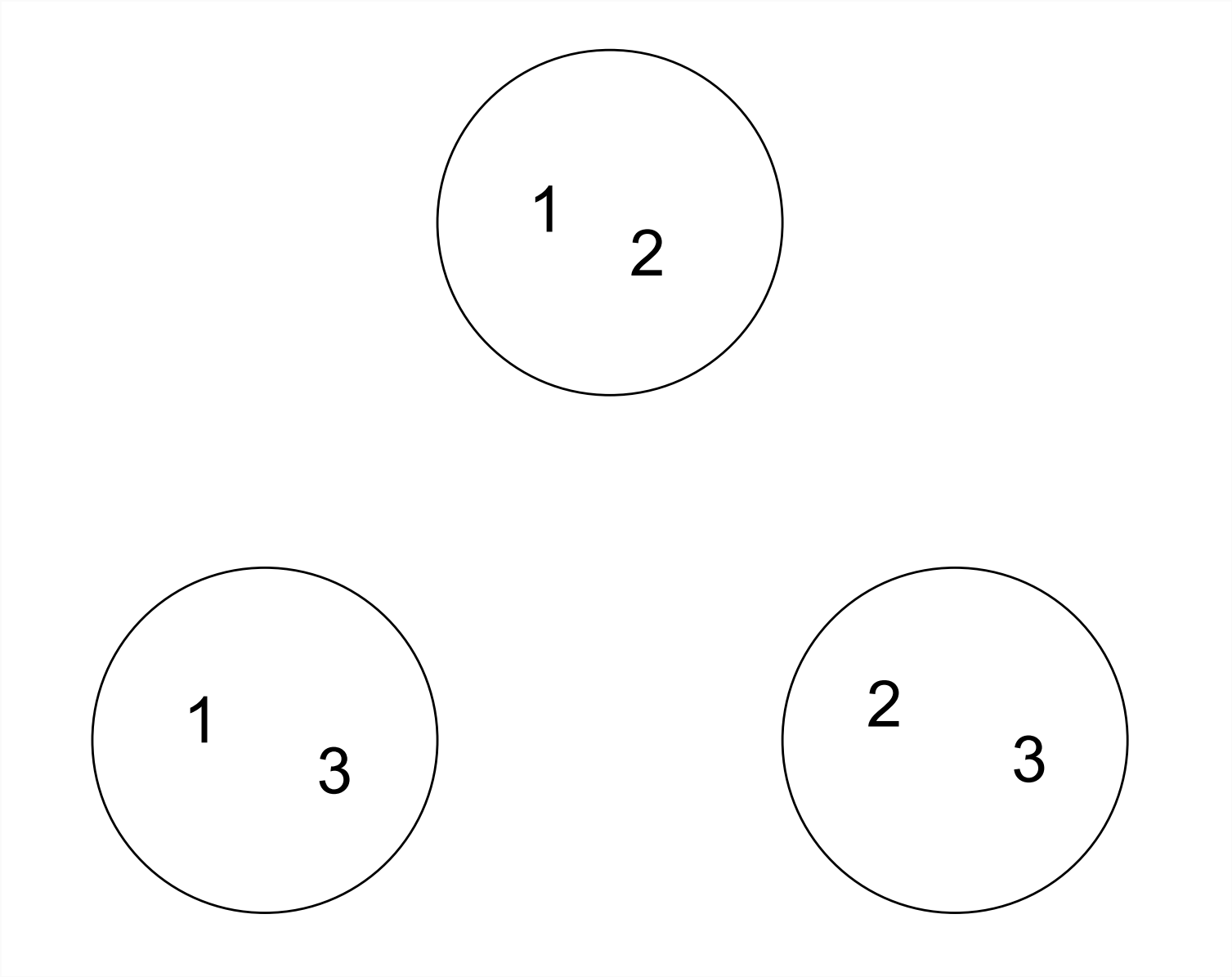
- Look for numerical patterns in that case and attempt to generalize and/or look for explanations of those patterns e.g., $s + (s-1)^2 = s^2 - s + 1$ cards given s symbols per card.
- Analyze the Spot It!^{TM} and Spot It! Junior^{TM} games and look for patterns.

When I ran this session at my Math Teachers' Circle, this is the direction I started with. I pivoted to projective geometry when we hit questions we couldn't answer and answers we couldn't explain.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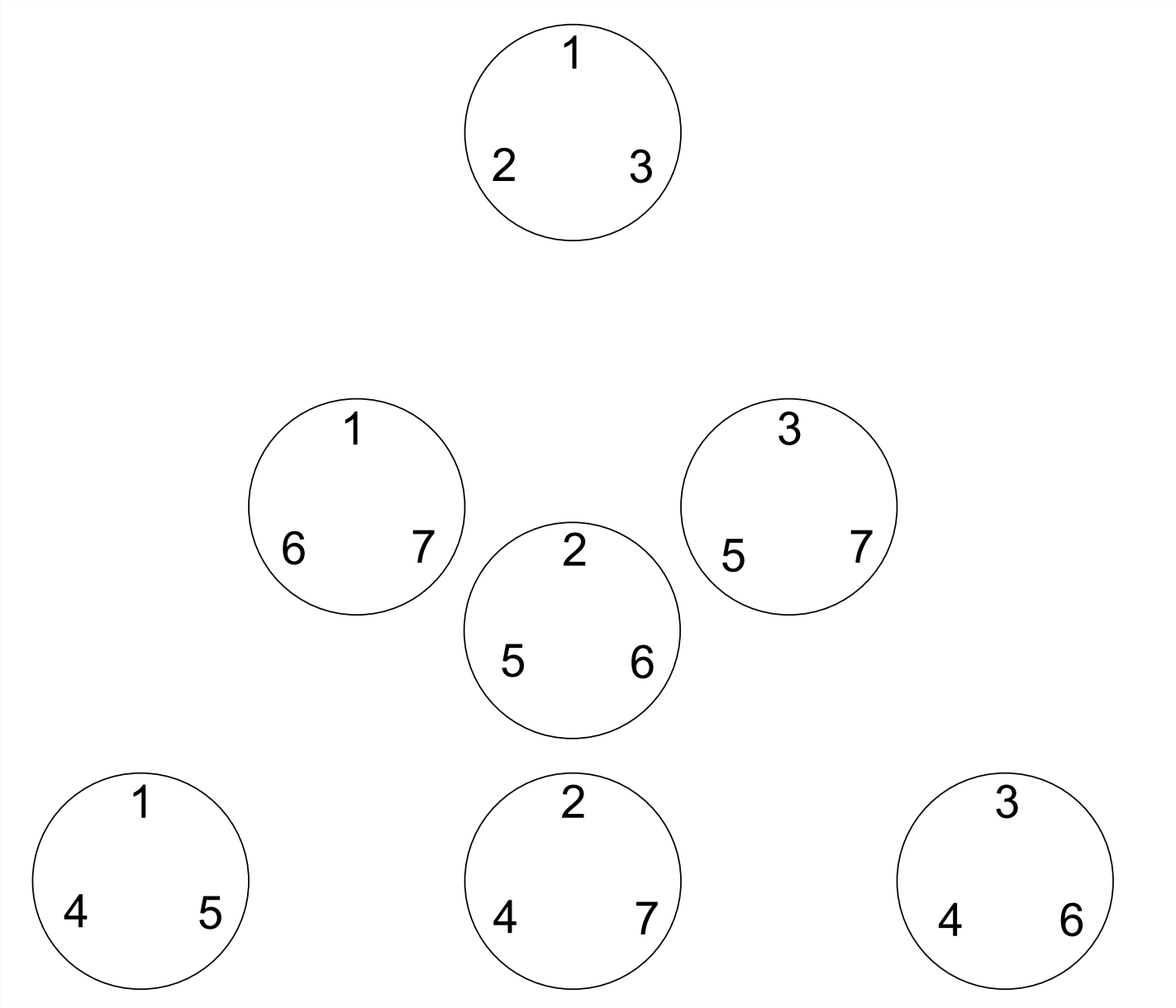
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Game Rules as Mathematical Axioms

Problem Solving Theme: Modeling / Using Precision

Question

What makes a good game? What makes Spot It! fun?

Game Rules as Mathematical Axioms

Problem Solving Theme: Modeling / Using Precision

Question

What makes a good game? What makes Spot It! fun?

- Given any two cards, there is always a match between them. (Function)
- Each card has same number of symbols. (Fairness)
- Symbols are equally "popular". (Fairness)

Projective Geometry Axioms

- Any two points are on exactly one line
- Any two lines have exactly one point in common.
- Four points must exist such that no three of them are on the same line.

Question

How do these axioms relate to Spot It!^{TM}?

Spot It!^{TM} Axioms

Definition

We define a **point** to be a Spot It!^{TM} card and a **line** to be a set of cards that contain a common symbol.

Spot It!^{TM} Axioms

Definition

We define a **point** to be a Spot It!^{TM} card and a **line** to be a set of cards that contain a common symbol.

- Any two cards must must have exactly one symbol in common (are on a "line").
- For any pair of symbols, there is a card which contains those two symbols.
- All cards contain the same number of symbols.
- There is at least one card.
- Every card contains at least three symbols.
- Every symbol is on at least three cards.

Deeper Questions

Question

Can you make a Spot It!^{TM} deck with 7 symbols on every card?

Deeper Questions

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Question

What about 9, 10, 11, 12, 13, ... symbols per card?

Secret Code

For those interested/skilled with computer programming there are some interesting directions to investigate Spot It!^{TM}

- Write a program that generates a deck of Spot It!^{TM} cards with a given number of symbols per card.
- Count the number of possible Spot It!^{TM} "decks" that can be made with a particular bank of symbols.

wrapping up...

Thanks For Your Attention!

To see any of my materials contact me at Tom.Clark@dordt.edu.

References

1. Clark, Thomas J. and Jongsma, Cal. *Analyzing Unique-Matching Games Using Elementary Mathematics*. (Forthcoming)
2. Dietz, Donna A. *Spot It! Solitaire*. `http://www.donnadietz.com/Dietz_SpotItArticle.pdf`
3. Lew. *Spot It!...A Combinatoric Cornucopia*. `http://www.lews-garage.org/spot-it-a-combinatoric-cornucopia/`
4. Sengupta, Deepu. *A Mathematical Analysis of Spot It!*. `http://www.mathteacherscircle.org/assets/legacy/resources/materials/DSenguptaSpotIt.pdf`