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There Really Are No Contradictions: A Response

Calvin Jongsma
Dordt College, calvin.jongsma@dordt.edu

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Abstract
Response to "There are no Contradictions" by T.G. Ammon in *The College Mathematics Journal,* Vol. 31, No. 1 (Jan., 2000), pp. 48-49 which was part of the "Fallacies, Flaws, and Flimflam" column edited by Ed Barbeau of the Department of Mathematics at the University of Toronto.

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The January 2000 issue of *CMJ* (31 (2000) 48-49) contained an article with the title *There are no contradictions* by T.G. Ammon. This drew the following comment from Calvin Jongma of Dordt College in Sioux Center, IA.

In the article, *There are no contradictions*, there really are no contradictions. Just a paradoxical conclusion generated by a confusion regarding what constitutes a proof or an argument. This confusion results from blurring the distinction between logical implication and deducibility. These are notions mathematical logicians have adequately distinguished for most of the twentieth century, but which many working mathematicians and students of mathematics may be less familiar with.

To explain the hoax further (that Aristotelian logic cannot generate any results from contradictory sentences), we will distinguish two senses of "argument". In one sense, an argument is merely a list of premises with a conclusion that logically follows from them. We will call this a premise-conclusion argument. Such a premise-conclusion argument is valid if the premises logically entail the conclusion, i.e., if it is impossible for the conclusion to be false while the premises are true. Here implication is the issue, not deducibility.

However, we usually mean by an argument something more than the bare-bones premise-conclusion sequence. In the ordinary sense of the term, an argument is a sequence of statements in which the conclusion is actually deduced from the premises in a step by step fashion. We will call this more normal sense of argument a proof or deduction. In a formal logical setting, each line of a deduction must be generated via stipulated rules of inference, each of which is sound (preserves truth). The nature and conclusive character of a deduction is determined both by the particular logic being used and also by the deduction system (set of inference rules) that are chosen for deriving conclusions from premises. In this second sense of an argument, genuine arguing must take place to demonstrate the logical connection holding between the premises and the conclusion. The former notion of a premise-conclusion argument does not involve any argumentation, only logical relations among sentences and connections between their potential truth values. Premise-conclusion arguments are exactly what require proof. A deduction of a conclusion from its premises using sound rules of inference is the way mathematicians establish validity.

Now, in the premise-conclusion sense of argument, where logical consequences are the issue rather than deductive consequences, contradictory categorical sentences X and X_{op} in Aristotelian Logic obviously imply any statement Z that can be stated in the language, just as is the case for Sentential or Propositional Logic. Every time X and X_{op} are both true (never), so is Z. The assertion in the article that they do not, therefore, is simply false.

It is true, though, that one cannot deduce anything from a syllogism whose premises are contradictories, for the simple reason that, strictly speaking, syllogistic reasoning requires three distinct terms. (If this requirement is relaxed, however, so that only two distinct terms occur, at least the contradictory argument "Some *S* are not *S*" can be deduced using the Aristotelian syllogistic form *A O O* - 2 on the premises *X*: "All *S* are *P*" and X_{op}: "Some *S* are not *P*", even if an arbitrary categorical sentence *Z* cannot be generated.) This conclusion about the limitations of strictly syllogistic reasoning should not surprise us; nothing much can be deduced from *P&~ P* in Sentential Logic either if the rules of inference are restricted to, say, the basic rules covering conditionals. Proving *Z* and ~*Z* from *P&~ P* requires rules for negation and conjunction. Similarly, in Aristotelian Logic, a Reductio Ad Absurdum rule based on the Square of Opposition can be introduced, and then any categorical conclusion *Z* and its contradictory Z_{op} can be deduced from the given premises. This is not a syllogistic rule, but then neither are the standard immediate inference rules known as Conversion and Subalternation, which are required to complete the deduction system for Aristotelian Logic.

So if we think about arguments in terms of logical implication, contradictories do imply anything whatsoever, while if we think about them in terms of deductions, they fail to deduce anything only if the rules fail to handle proofs by contradiction. This is true for both Sentential Logic and Aristotelian Logic. Contradictions are fully as constructive in a robust version of Aristotelian Logic as they are in Sentential Logic. Where Aristotelian Logic is limited is in its range of expressibility, not in its ability to imply or deduce logical consequences.