Is a Flipped Classroom in a High School Geometry Class Effective?

Charles Dirks

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Document Type
Thesis

Degree Name
Master of Education (MEd)

Department
Graduate Education

First Advisor
Patricia C. Kornelis

Keywords
Master of Education, thesis, flipped classrooms, lectures and lecturing, secondary schools, geometry

Subject Categories
Curriculum and Instruction | Education

Comments
Action Research Report Submitted in Partial Fulfillment of the Requirements for the Degree of Masters of Education

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By

Charles Dirks

B.S. Kansas State University 2005

Action Research Report
Submitted in Partial Fulfillment
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Degree of Masters of Education

Department of Education
Dordt College
Sioux Center, Iowa
July, 2018
Is a Flipped Classroom in a High School Geometry Class Effective?

by

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Abstract

This action research study investigated the effects of the flipped classroom on a high school geometry class in a rural setting. Thirty-nine high school students ranging from ninth to the eleventh grade participated in the study. These students were already divided into three separate classes. Two of the geometry classes became the experimental group and received a new flipped model of teaching. The control group was one geometry class and was given the same lecture style of teaching that the teacher had used for the past three years. Students were given a pre- and post-test to determine if after receiving the flipped style of teaching they retained more material than the traditional lecture style. The results of the study showed that there was no significant difference in test scores.
In *Best Practices* (2012), a comparison to doctors and educators is drawn. The premise is simple: doctors are constantly searching for best practices so they can help society live longer and have a better quality of life. These best practices, often in the form of new techniques and methods, are increasingly linked to advances in technology. On the other hand, even though the face of education seems to be changing, numerous educators still seem to hang on to the old, tried and true methods they were taught because those methods worked for them when they were students. The argument is that if these methods worked for them, then they should still work in today’s educational environment.

Are people still be willing to go to doctors today that say, “I practice medicine exactly the same way today as I did thirty years ago. I haven’t changed a thing. I don’t pay any attention to all the newfangled mumbo-jumbo-MRIs, vaccines, antibiotics, and such” (Zemelman, Daniels, & Hyde, 2012, p. 2)? That is exactly what teachers are communicating to students and parents when they are not willing to change because their methods have worked for years. Teachers who do not do anything different no matter how much technology has advanced could be missing an opportunity in helping the students of today.

The problem with this “it worked before, it should work today” logic is that older methods in teaching, especially mathematics, are not as effective as once thought. Many students are now struggling not only to grasp the concepts, but also to retain knowledge. The loss of retention is verified by the Program for International Student Assessment (PISA), which compared multiple countries’ fifteen-year-old students in English, science, and mathematics. According to Organization for Economic Co-operation and
Flipped Classroom

Development, the United States scored a little above average in English and science, but was 20 points below the worldwide average in mathematics (Carr, 2016). This overall score in mathematics ranked the United States 36th in Math. Carr (2016) also pointed out “Of particular concern is that we also have a higher percentage of students who score in the lowest performance levels, compared to the OECD average, and a lower percentage of top math performances” (para 7). Not only are the United States’ top students struggling to keep up with other countries’ top students, but more of the United States’ average students are moving towards the lower levels and falling even farther behind.

Educators facing this decline in nationwide math scores are looking for answers. According to Willis (1985), technology has changed how the younger generation processes information, how they relate to others, and even how they view their place in the world (p. 20). The way the world operates has changed so much over the past 20 years, but styles and methodologies of teaching students have not progressed. Weiss (2004) attributed a dip in scores to a passive learning experience most students receive in the classroom. Best practices are ones that make students interact with math and relate it back to prior knowledge (p. 26). Since students’ mathematical retention has dipped, a change of approach in how students are taught may be required to help catch up with the rest of the world.

**Statement of the Problem**

Schools are starting to realize that the United States is falling behind other nations in mathematics. Some teachers are finding new ways to bring students into the center of the classroom and to help them in the retention of mathematical ideas and concepts. The flipped classroom model is one of the ideas that has started to show up in some schools.
Even though it is a newer concept to teaching, going back only about fifteen to twenty years, it seems to be a strategy that has gained more attention as a powerful teaching tool with about 80% of teachers at one point flipping at least one lesson (Weston, 2017, para. 3).

The concept of the flipped classroom is a pedagogy-first approach to teaching. In this approach, in-class time is re-purposed for inquiry, application, and assessment in order to better meet the needs of individual learners. Students gain control of the learning process through studying course materials outside of class, using reading, pre-recorded video lectures, using technology, or research assignments. These lower level thinking skills, remembering and understanding, are done before class. During class time, instructors facilitate the learning process by helping students work though course materials that cover the higher level thinking skills: applying, analyzing, evaluating, and creating (Zamzami & Halili, 2016, p. 316). The principle behind flipping the classroom is helpful to students. Students most often experience frustration with math homework because they have forgotten the details and procedures given in class, therefore leading them to be less motivated to stay engaged in the classroom. A study by Zamzami and Halili (2016) found “that the flipped classroom has been successfully practiced to better engage student in learning various subjects. In contrast, the class without flipping or traditional class tends to produce disengaged learning environment because this conventional learning model has some problems” (p. 329).

When using the flipped model of instruction, time that was previously reserved for homework now becomes the time that students watch videos and take notes about key concepts that will be reviewed and covered more in-depth the next day in class. Can the
flipped classroom model be an effective method for teaching math and will it lead to increase in student retention of mathematical concepts? This research study sought to explore if the flipped model is a more effective method of teaching mathematical concepts to high school math students.

**Research Question**

1. Is the flipped classroom a more effective way to help high school geometry students retain mathematical concepts and procedures than traditional lecture-style teaching?

**Definition of Terms**

A basic understanding of the key terms used in this study is essential in order to generalize the results to the teaching practice. The terms defined here are the author’s unless otherwise stated.

*Flipped Classroom/Model:* Zainuddin and Halili (2016) best define the flipped classroom or model by stating, “the lower levels [of blooms taxonomy] are presented before class through recorded lectures and video. Readings, simulations, and other materials also provide this foundational support for learning so that in-class time can be spent working on higher levels of learning from application to evaluation” (p. 316).

*OECD:* The Organization for Economic Cooperation and Development is 34 democracies with market economies that work together to promote economic growth, prosperity, and sustainable development.

*PISA:* The Programme for International Student Assessment is a test given to students across the world used to compare student achievement. PISA is sponsored by the OECD.
ESL: English as a Second Language. These are all students whose first language is something other than English.

IEP: Individual Educational Plan is a program that is usually set up for special education students in order to help them in the regular education classroom.

Review of Literature

The flipped model is an appropriate one for today’s students and how they learn. Today’s college students have grown up immersed in technology. As such, they expect to be able to access information on demand, and they arrive on college campuses ready to engage information in new ways (Ford, Burns, Mitch, & Gomez, 2012, p. 191). Students are spending increasing amounts of time interacting with social media. A study done by the Kaiser Family Foundation in 2010 discovered that

[O]ver the past five years, young people have increased the amount of time they spend consuming media by an hour and seventeen minutes daily, from 6:21 to 7:38—almost the amount of time most adults spend at work each day, except that young people use media seven days a week instead of five. (Rideout, For, & Roberts, 2010, p. 2)

Also on the increase is the amount of technology in the schools. A national report conducted each year by IESD and STEM Market Impact, based on survey responses from 332 district leaders found that of the district leaders who responded, 71 percent said that a quarter or more of their schools have adopted mobile technology, which is up from 60 percent in 2013 (Logan, 2013). Not only do students have more access to social media and online study tools at school through a one-to-one initiative, but students also own devices such as tablets, laptops, and even smartphones. In fact, a study done by Pearson
(2015) showed that only 3% of all students in 2015 did not use or have access to technology.

Since students’ availability of technology has grown, teachers need to find new ways to capture some of this screen time. Even if teachers could find a way to tap into only a tenth of this time that students spend using technology, then instructional time could increase by almost five hours each week. This instruction time would be valuable for introducing and reviewing concepts. The flipped classroom is one method that could use screen time to a teacher’s benefit.

The flipped classroom has a proven history with multiple teachers using this concept of videos to help students gain knowledge while using classroom time to expand that knowledge and get students active with the learning process. Ziegelmeier and Topaz (2015) examined two identical classes taught by the same instructor and compared not only test scores but also the number of quizzes finished by both groups. They noticed that there was not a significant difference in scores but did notice, “students seemed to be more engaged during the flipped class” (p. 856). In a second study, performed in a secondary math classroom, two classes of Algebra I students were examined. One class was taught using a flipped model of instruction and the other used the traditional, direct instruction, methods. After collecting data over a seven-week period, the researchers noted “the difference among performance measures between the traditional and flipped classrooms can be described as insignificant”; however, “students were more actively involved in the flipped classroom than the traditional environment” (Clark, 2015, p. 103). These studies show a trend that when a flipped model is used, student engagement is increased.
Estes, Ingram, and Liu (2014) reviewed research, practice, and technologies that were used in the classroom, and found that “flipped learning in higher education is growing rapidly, and nine of ten teachers who responded to a Sophia & Flipped Learning Network Survey 2014 reported improvements in student engagement” (para. 35). One reason for this rise in flipped learning seemed to be student driven. Davis and Summers (2014) conducted a survey and when asked if they agree with the statement “I believe that learning experiences that simulates ‘doing the real thing’ are more effective than traditional methodologies,” the results showed the 90 percent of the people agreed with this statement (p. 6).

In describing traditional teaching methods, the Friday Institute for Educational Innovation stated, “students spend 90 percent of their time absorbing lectures and only 10 percent of it applying what they learned” (Ryals, 2011, para. 12). With the flipped model, more time is spent in the applying and less on the passively listening to lectures.

While the research of Graziano and Hall (2017) indicates that student engagement increases within the flipped classroom, the research on the impact on students’ achievement in the flipped classroom is less conclusive. Graziano et al (2017) compared two Algebra I classes, and focused on the English Language Learners (ELL) students. “Results indicate no statistical significant mean difference in academic performance from students enrolled in Algebra I with flipped instruction compared to students enrolled in the same” (p. 10). Other studies showed some of the same results. A study by Sparks (2013) had mixed results finding that “[f]lipping the classroom did improve the test scores for 14% of the students” but also found “81.5% of students showed no improvements and 3.7% showed lower test scores using the flipped classroom method”
Even though the scores of some students lowered, overall there was a 14% increase which indicated a substantial growth in achievement for the majority of students.

In another study done by Unal and Unal (2017), researchers examined the results of 16 teachers’ flipped model experiences at different grade levels. After designing the units pre- and post-tests, these teachers switched their instruction to a flipped model. The results of this study showed that 10 teachers’ students scored significantly higher while using the flipped model, while one teacher’s students scored significantly higher using the traditional model of instruction. The remaining five teachers’ students showed no significant difference in their scores. In addition, the Unal and Unal (2017) study pointed out that the “results of this study showed significant learning gain differences mostly in favor of the flipped classrooms because it promotes active learning, which requires students to solve problems using what they learned before class” (p. 157).

Active learning seems to be a common theme throughout other studies as well. Clarke (2015) conducted a research study on a secondary mathematics classroom. He noted that “during the flipped classroom, the students witnessed an increase in their classroom participation and communication, thus promoting a student-centered classroom environment conducive to learning and success” (p. 103).

In order to achieve this active learning, teachers need to be aware of how effectively they use their time in the classroom. This change of mindset can be difficult for lecture-style teachers as Graziano et al (2017) pointed out, “a challenge for novice teachers who flip the classroom is how to effectively use class time, which may be especially challenging for teachers who are accustomed to direct instruction” (p. 14). However, if teachers are able to utilize their time with the flipped method, Clark’s (2015)
research showed that “[w]hen compared to the traditional environment, the student participants argued there was better use of class time with the flipped model of instruction” (p. 104).

**Methods**

**Participants**

The participants in this study were students from a 3A school in rural, western Kansas with average enrollment between 180-200 in grades nine through twelve. All participants in the study were in one of three regular education high school geometry classes taught by the same teacher.

The two smaller classes, one class with eight students and one with ten, were the experimental group which participated in the flipped model of instruction. The third class with 19 students was the control classroom which received traditional instruction. Random selection was not used in this study as students were already assigned to these three geometry classes. Both groups had a mean age of fifteen years old, and each group consisted of 30% Hispanic population with the rest being categorized as Caucasian. The experimental group included one student in the ESL program and three students that had an IEP. The control group included one student with an IEP.

**Materials**

The instrument used for this research study to measure student achievement was a final unit test, in Appendix A, given each year to high school geometry students upon completion of this geometry unit. The test was created by the high school math department with each question vetted by the four math teachers to determine validity and clarity of questions. All of the test questions were multiple choice to ensure unbiased
grading. This test was looked at before the nine-week mark to ensure that all materials were covered, but for the purpose of this study, the test was reviewed before the beginning of the semester and all materials on the test were covered. In both the pre-test and post-test, each student received the same thirty questions, but for every test the questions were in a different order. Schoology, the schools learning management system, was used to create, administer, and grade the test. Students were allowed to take the test during the designated hour-long class period, and if they did not finish, they were given an extra half-hour after school with the teacher present. All students were present the day of testing, and no makeup testing was necessary.

**Research Design**

At the start of a pre-determined unit, all classes were given the same 30-question, multiple-choice test. The test was given during the first day of the new unit without any previous instruction. The test was given to establish each student’s baseline and knowledge of the unit. The three geometry classes were designated as control and experimental groups. The control group, one class with 19 students, received the lecture-style teaching used in the past. The experimental group, made up of two classes with 18 students total, were taught using the flipped classroom method. Each group worked on the unit for the same amount of time: 14 school days. Each group was given the same 30-question, multiple-choice post-test (same test that was administered at the start of the unit.) Scores were compared to see if the amount of improvement was significant in one style of teaching over the other.
Procedure

On the first day of the unit, all participants took a 30-question pretest using Schoology. The scores were not shared with the participants, and not used to change any instructional methods for the teacher. The purpose of the pre-test was to set a baseline for the action research.

After the pretest was administered, the experimental group was instructed in how the flipped model of instruction would work. The students were shown how to use screencast-o-matic for any videos they were asked to make. They were also shown how the new format of class would work with watching videos at home. In order to ensure students were watching videos daily, note taking was required. Students showed the instructor their notes from the previous night, and a grade was taken for completed notes. In the experimental group, students were assigned to groups each day. On some days, they were grouped by ability and on other days the grouping was random. During each class period the experimental group usually had two to three stations to complete. Most of the time stations included discussion time with the instructor, worksheet skills practice, challenge problems, real-world application problems, fifteen-minute research projects, and video-share time. Students spent no more than twenty-five minutes at any one station depending on the level of difficulty. The only homework that students took home was to watch the next video. Students did not have a video every night, but if new material was to be covered, a video was made to help students preview the topic. Videos were never longer than 15 minutes. The videos were posted to Schoology so that students were able to access them at school or at home. If students did not have access to the Internet at home, they could download the videos before leaving school.
The participants in the control group were taught using traditional lectures that were created three years ago. These lectures have been tweaked each year, and have been improved each year by the same instructor. Each day the instructor spent the first twenty-five to thirty-five minutes lecturing about the material for the day, and then questions and homework were given to work on and finish at home.

All of the assignments for both the experimental and control groups were issued through Schoology. Unit test and quizzes were given in the same method, usually paper and pencil or online through Schoology. At the end of the unit, students were given the final exam, which was the same as the pre-test. Schoology graded all tests; all questions were multiple-choice response therefore keeping human error or bias out of the teacher’s hands.

Results

Data Analysis

This study’s purpose was to determine if the flipped model of teaching math was a more effective way of teaching than the lecture style that has been used in the past. In order to determine if the flipped method was better than the lecture style, data was gathered at the start and end of the unit through the 30-question multiple-choice test. The study compared the average difference between the two groups to determine if the level of retention was greater in the flipped model. For measurement purposes, the researcher used a two-sample t-test to help determine if there was a significant difference to the .05 levels between the two averages.
Findings

To determine if the flipped model was more effective than the lecture style, the difference in post-test to pre-test was found for each individual student. Those differences were averaged in each group in order to compare overall average improvement. When looking at the averages in Figure 1, it was clear to see that both groups had about the same amount of prior knowledge at the beginning of the unit. The control group showed a slightly higher level of prior knowledge. At the end of the unit, the flipped classroom did score better on the post-test overall, but only by one percentage point which was not enough to conclude there is a significant difference between the two groups.

![Figure 1: Bar graph showing the comparison of pre- and post-test average scores.](image)

Next, the researcher calculated the average amount of improvement from each student in order to come up with a mean difference for the experimental and control group. Once again, both groups were close to each other in the amount of improvement that they showed over the unit. Figure 2 illustrates the average amount of improvement.
After compiling all the data, a two sample t-test was performed to see if the difference in average improvement was significant. For this test a .05 level of significance was used. A t-statistic of 1.261 was found, with a critical value of 1.703. Since the t-statistic was less than the critical value the researcher concluded that there was not a significant difference between the two groups in overall growth. The researcher sought to answer the research question, is the flipped classroom an effective way to help high school geometry students retain mathematical concepts and procedures? The results of this study indicate that the answer to the research question is inconclusive.

An interesting finding that resulted in this from this study was the amount of change in students that had an overall class grade of less than a B, less than 80%, before the unit started. Both groups had only a few students in this range, but there was a difference between the two groups with the experimental group out preforming their peers in the control group by 11%. Figure 3 illustrates this finding.
Discussion

Summary

When looking into the effectiveness of the flipped classroom, it would be easy to assume that flipped learning is simply making videos for students to watch at home and worksheets for them to work on when they are in the classroom. Instead, it is a methodology that front loads the content for the students and sets them up to succeed in the classroom. Even though the study showed no significant difference in test scores between the experimental group and control group, some increases in scores, especially with the students that had little success before, showed that there could be something to this technique that helps the lower level students while still keeping the higher level students engaged. As technology continues to change, more study and research will be necessary to discover if the flipped model is a viable technique for educational instruction.

Figure 3: The average improvement for students who had less than 80% in the class before the unit started.
Study Limitations

A few limitations need to be considered in this study. First is the length of the study. As a result of only covering one-unit students were exposed to the flipped learning style for a brief period of time. If the study was continued over a longer time period, the expectation would be a higher level of retention. This improvement of retention would be due to the increase of proficiency with the use of technology and more efficient use of classroom time.

The second limitation would be the sample size in this study. With both groups having less than 30 participants, the average mean scores can easily be skewed. A larger sample would help show the effect of the flipped classroom on a wider range of students with different learning styles and prior knowledge.

The final limitation to this study could be the “newness factor.” Seeing something new and doing something different in school could be what connected with the experimental group and made them more excited about coming to math class. However, the newness of a flipped classroom could wear off and students may not be as excited and willing to participate. Overall student improvement could be great if the students were given time to adjust to newness of this different teaching style. In this study, none of the students had ever been exposed to a flipped classroom, and the first few weeks required an obvious adjustment period. Overcoming these limitation would be helpful in coming to a stronger conclusion about whether the flipped model is a better methodology than the lecture style model.
Conclusion

Although the study did not show a significant difference in test scores, it seems that some students, especially the lower achieving, respond positively to the flipped model. One of the reasons could be the student-to-student interactions, and the teacher and student interactions seemed to be more numerous when using the flipped model as opposed to the lecture-style model. These interactions made the students more receptive to ideas and allowed them to learn together. Even the higher achieving students seemed to be pushed more and struggled more to learn the material, which could be more beneficial to the learning process. This researcher concluded that further study is required to see if the flipped model does, in fact, increase retention to all students within the context of the geometry classroom.
References


Appendix A

Pre-Post Test

Find $x$. Assume that segments that appear tangent are tangent.

a. 5  
b. 7  
c. 13  
d. 11
\( NA \equiv PA, \ MO \perp NA, \ RO \perp PA, \ MO = 3 \text{ ft} \)

What is \( PO? \)

a. 9 ft  
b. 6 ft  
c. 3 ft  
d. 1.5 ft

In \( \odot F \), \( \angle CFD \equiv \angle DFE \), \( m\angle BFA = 8x \), \( m\angle AFE = 5x + 11 \), and \( \overline{BE} \) and \( \overline{AC} \) are diameters.

Find \( m \angle DC \).

a. 56  
b. 52  
c. 47  
d. 51
$AB = 20$, $BC = 10$, and $CD = 6$

(a) 50  (b) 33.33  (c) 22.67  (d) 44

(a) 144  (b) 14.2  (c) 18  (d) 4.5
Find the diameter of the circle for $BC = 13$ and $DC = 28$. Round to the nearest tenth. (The diagram is not drawn to scale.)

![Diagram of a circle with points C, B, O, A, and D, where C and D are outside the circle, and B and O are on the circle.]

a. 10.8  
   b. 73.3  
   c. 60.3  
   d. 47.3

Find the measure of value of $AB$ for $m\angle P = 22$. (The figure is not drawn to scale.)

![Diagram of a circle with points A, L, B, and P, where A and B are on the circle, and L and P are outside the circle.]

a. 158  
   b. 44  
   c. 11  
   d. 316

The circles are congruent. Given $\overline{HJ} \equiv \overline{LM}$, what can you conclude from the diagram?


a. $\overline{HJ} \equiv \overline{LM}$  
   b. $\angle JOH \equiv \angle MPL$  
   c. $\angle JOH \equiv \angle MPL$ and $\overline{HJ} \equiv \overline{LM}$  
   d. none of these
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Diagram 1:
- a. 213.3
- b. 21.2
- c. 24.7
- d. 14.6

Diagram 2:
- a. 34.5
- b. 42
- c. 69
- d. 21
Find $\angle BAC$. (The figure is not drawn to scale.)

![Diagram of circle with angles and point O]

a. 39  
b. 102  
c. 51  
d. 156

Find $x$. Assume that segments that appear tangent are tangent.

![Diagram of circle with segments and point O]

a. 14  
b. 10  
c. 16  
d. 28
\[ m \angle P = 29 \]

\[ \begin{align*}
    a. & \quad 119 \\ 
    b. & \quad 61 \\ 
    c. & \quad 30.5 \\ 
    d. & \quad 58 \\
\end{align*} \]

\[ m \angle O = 124 \]

\[ \begin{align*}
    a. & \quad 304 \\ 
    b. & \quad 248 \\ 
    c. & \quad 56 \\ 
    d. & \quad 62 \\
\end{align*} \]
Find the value of $x$ for $m\overline{AB} = 36$ and $m\overline{CD} = 34$. (The figure is not drawn to scale.)

a. 35  
b. 70  
c. 53  
d. 2
Find \( m\angle D \) for \( m\angle B = 63 \). (The figure is not drawn to scale.)

\[
\begin{align*}
A & \quad B \\
D & \quad C \\
\end{align*}
\]

\begin{align*}
a. \ 54 & \quad b. \ 58.5 & \quad c. \ 108 & \quad d. \ 117
\end{align*}

\[
\begin{align*}
5 & \quad x \\
15
\end{align*}
\]

\begin{align*}
a. \ 10 & \quad b. \ 7 & \quad c. \ 8.7 & \quad d. \ 5
\end{align*}
In \( \odot D \), \( AB \cong CB \) and \( m \text{ arc } CE = 54 \). Find \( m \angle BCE \).

![Diagram of circle with points A, B, C, D, and E]

- a. 108
- b. 109
- c. 81
- d. 113

The radius of circle \( O \) is 22, and \( OC = 6 \). Find \( AB \). Round to the nearest tenth, if necessary. (The figure is not drawn to scale.)

![Diagram of circle with right triangle AOB]

- a. 26.9
- b. 42.3
- c. 21.2
- d. 45.6
If $m\angle 1 = 3x + 2$, $m\angle 2 = 5x$, find $m\angle 1$.

a. 58  

b. 37  

c. 35  

d. 55
Find $x$. Assume that segments that appear tangent are tangent.

Find the measure of $\angle BAC$. (The figure is not drawn to scale.)
Given that $\angle DAB$ and $\angle DCB$ are right angles and $m\angle BDC = 59^\circ$, what is $m\overline{CAD}$? (The figure is not drawn to scale.)

a. 236    b. 298    c. 354    d. 357

In $\odot A$, $\overline{AC} \cong \overline{AF}$ and $AE = 29$.

Find $m\overline{EG}$.

a. 40    b. 42    c. 29    d. 41
\( m \angle R = 31 \). Find \( m \angle O \). (The figure is not drawn to scale.)

\[
\begin{array}{c}
\text{a. 59} \\
\text{b. 31} \\
\text{c. 149} \\
\text{d. 62}
\end{array}
\]

\( \widehat{DE} = 117 \) and \( \widehat{BC} = 70 \). Find \( m \angle A \). (The figure is not drawn to scale.)

\[
\begin{array}{c}
\text{a. 47} \\
\text{b. 93.5} \\
\text{c. 82} \\
\text{d. 23.5}
\end{array}
\]
If $m\overline{BY} = 28$, what is $m\angle YAC$? (The figure is not drawn to scale.)

- a. 152
- b. 76
- c. 56
- d. 124