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Meaningful Instruction of Basic Multiplication Facts: Applying Constructivist Concepts to Basic Fact Acquisition

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Abstract

In the United States, third grade students are expected to be fluent in the 100 basic math multiplication facts by the end of the school year. This action research project investigated the correlation between the use of constructivist principles for learning the 100 basic math multiplication facts and its impact on student understanding of mathematical reasoning concepts. The study involved twenty-two students in a midwest suburban third grade Christian school classroom for a 30-day period. An average of one hour a day was spent in math class with varied use of individual, partner, small group and large group work and instruction throughout the weeks. In the study basic multiplication facts were introduced through the use of strategies that included exploration, manipulative objects, guided reinvention, developmental pacing, interactive dialogue and relational understanding with the goal of building fluency in the facts and also impact understanding in other areas of math reasoning. The pretest and posttest were the Fall and Winter MAP Math tests respectively. The results of the study indicated a positive impact in overall math scores for students who scored at or below 80th percentile on the Fall MAP math test and a positive impact for males in overall algebraic reasoning.

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Abstract

In the United States, third grade students are expected to be fluent in the 100 basic math multiplication facts by the end of the school year. This action research project investigated the correlation between the use of constructivist principles for learning the 100 basic math multiplication facts and its impact on student understanding of mathematical reasoning concepts. The study involved twenty-two students in a midwest suburban third grade Christian school classroom for a 30-day period. An average of one hour a day was spent in math class with varied use of individual, partner, small group and large group work and instruction throughout the weeks. In the study basic multiplication facts were introduced through the use of strategies that included exploration, manipulative objects, guided reinvention, developmental pacing, interactive dialogue and relational understanding with the goal of building fluency in the facts and also impact understanding in other areas of math reasoning. The pretest and posttest were the Fall and Winter MAP Math tests respectively. The results of the study indicated a positive impact in overall math scores for students who scored at or below 80th percentile on the Fall MAP math test and a positive impact for males in overall algebraic reasoning.

Keywords: basic multiplication facts, constructivist

Learning basic multiplication facts has traditionally been a rite of passage in the intermediate grades of school. In the United States, by the end of the school year, third-grade students must be able to “know from memory all products of two one-digit numbers” (CCSSI 2010, p. 23). To further accentuate the need for mastery of math facts, in 2001, the National Research Council stated that fluent computation is a vital part of mathematical competency (Kilpatrick, Stafford, & Findell). Parallels in math have been made to S. J. Samuels’ automatic processing theory of reading, noting that just as fluent readers can devote more cognitive energy to greater reading skills, so students who use minimal thought to make basic fact calculations can focus their energy to greater complexities in mathematics (Singer-Dudek & Greer, 2005). Though fluency in basic facts does not equate with fluency in other mathematical processes, it is likely that students who struggle with the facts will have difficulty making broader gains in mathematics (Nelson, Parker, & Zaslofsky, 2016). Building fluency in basic math facts is key to students’ further success in multiplication. Teachers need to be strategic in their teaching delivery, allowing for student self-exploration as well as clearly demonstrating advanced strategies to help the students move to the next level of complexity in understanding (Van de Walle, Karp, Lovin & Bay-Williams, 2014; Zhang, Xin, Harris & Ding, 2013).

Problem

Traditionally, basic math facts were learned by rote memorization. This involved a drill method, with the assumption that formal repetition would best lead to efficiency and thorough knowledge. Of greatest concern was speed and accuracy. However, drill did not not enhance students’ math reasoning skills and understanding (Baroody 1985; Baroody, 2006; Brownell, 1935; Chung, 2004; Gravemeijer & van Galen, 2003; Kling & Bay-Williams, 2014; Kling & Bay-Williams, 2015; Van de Walle et al., 2014).

The constructivist theory of learning supports the need for meaning-making in learning math facts. According to the constructivist theory, facts learned in isolation do not have an anchor to hold position in our brain. Relational understanding, though, forms a strong memory bond; by relating extant knowledge to new ideas, meaningful learning is constructed. Accomplishing this feat requires reflective thought as the past knowledge needs to be adjusted to assimilate the new information. (Seifert & Sutton, 2009; Van de Walle et al., 2014). Simply put, learning equals modification in thinking.

Basic math facts acquire meaning when related to earlier learned mathematical principles. By seeing numbers as compositions of other numbers, patterns are detected and the basic facts become building blocks to add on to the foundation of solid numeracy skills (Kling & Bay-Williams, 2015). Students need to retain basic fact information as a lifelong acquired skill. If the acquisition of basic math facts is left without connection to existing schema, retention is lost.

So how does the teacher provide an environment for meaningful interchange between basic math facts and students? By nature, humans look for connections and for meaning (Baroody, 2006; Graham, 2009). What if students - and teachers - saw the interconnectedness of the patterns of numbers embedded in creation? This takes basic facts out of the mere arithmetic category and applies them to the fuller realm of mathematical reasoning. This is vital for all students.

The picture that is emerging is one of thoughtful and meaningful instruction, resulting in students seeing facts in relationship with one another, rather than dozens of isolated facts. Learning basic math facts involves a network of connections that need to be made between the facts. This leads to more efficient fact recall and long-term retrieval capabilities. Derived facts have a key place in the development of students’ learning their multiplication facts. The scaffold and structure are in place for students to construct new knowledge, using what they already know and building on it to work toward more connections in learning (Baroody, 2006). Strategic teaching is needed to guide students in establishing connections amongst the 100 basic facts.

In order to determine what this strategic teaching involves, the researcher designed a study to address the third grade requirement that all students be fluent in their basic multiplication facts to 100. By applying the constructivist theory of learning, the researcher sought to establish a correlation between understanding and practicing the interconnectedness of the facts and greater overall mathematical reasoning gains. The interconnectedness of facts was practiced through exploration, manipulative materials, guided reinvention, developmental pacing, interactive dialogue and relational understanding.

Research Question

This research study sought to address the following question: Does applying the theory of constructivist learning to the strategic instruction of basic multiplication facts, through the use of strategies that include exploration, manipulative objects, guided reinvention, developmental pacing, interactive dialogue and relational understanding, have a significant impact on student understanding of mathematical reasoning concepts?

Definitions

For the purpose of this research study, the following definitions will be used. Unless otherwise noted, the definitions provided are those of the author.

- <u>Basic Multiplication Facts</u> - two whole number factors that result in a product up to 100.
- <u>Derived Facts</u> - facts found by using known multiplication basic facts to solve unknown basic multiplication facts.

- Fluency - procedural fluency is using appropriate strategy, flexibility, efficiency and accuracy when performing procedures (CCSS1 2010, p. 6).
- Foundational Facts - facts used to build derived facts. In multiplication foundational facts include x 1, x 2, x 5, and x 10 (Kling & Bay-Williams, 2014). These foundational facts are established prior to employing any derived fact strategies.
- Mathematical Reasoning - the ability to notice patterns (inductive reasoning), employ logic (deductive reasoning), apply properties (algebraic/symbolic reasoning) and interpret pictures and puzzles (geometric/spatial reasoning) (Rogness, 2008).

Summary

Third graders need to become fluent in their basic multiplication facts. Strategic teaching of basic multiplication facts, through application of the constructivist theory of learning, may ensure not only greater retention of math facts, but should lead to student gains in overall mathematical reasoning. Strategic teaching in this research study included the demonstration and student use of manipulative objects, guided reinvention, developmental pacing, interactive dialogue and relational understanding.

Literature Review

Becoming fluent in basic multiplication facts requires student understanding. Already in 1935 Brownell saw the need for meaning-making in basic fact instruction and proclaimed the value of developing number sense in children. He dubbed his ideas the “meaning” theory of arithmetic and made clear that learning basic facts was not an exercise of unrelated facts to be placed into one’s memory. Rather, he stated, “The ‘meaning’ theory conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes” (Brownell, 1935, p. 19). Furthermore, he argued that children should not be expected to simply copy the proficiency of the adult, but rather explore in their own developmental way. He saw learning basic facts as a slow process that should not be rushed. What may have appeared immature to adults, said Brownell (1935), resulted in a child's eventual understanding of the number combinations and their underlying relationships. For many years, Brownell's ideas were largely disregarded.

After studying how students learn new information, Piaget came to the conclusion that children were not blank slates to pour information into, but rather they created their own learning. It was through past schema that learners connected new information. When the new information contradicted with their extant knowledge, accommodations needed to be made to assimilate the new information into their repertoire. Prior knowledge was key to building new learning (Seifert and Sutton, 2009; Van de Walle, Karp, Lovin, Bay-Williams, 2014). Thus, expecting students to memorize facts by rote was a disservice. In order for meaningful learning to occur, the learner needed to actively seek meaning in new information. If the new information did not connect to prior knowledge, long term retention was questionable (Kling & Bay-Williams, 2015).

Adding to the constructivist theory, Vygotsky (1978) focused on the need for learners to work with others who were more knowledgeable. He believed that each learner had a personal zone of proximal development (ZPD). By receiving cues from more advanced learners, new connections could be made that would not be made without expert help. This social constructivist theory became known as the sociocultural theory (Seifert and Sutton, 2009; Van de Walle et al., 2014). Having qualified teachers and then having students engaged in conversations about the processes was believed to be key in this theory. The teacher was not the sole teacher in the classroom. Students learned from others and students also learned by putting their thought processes into coherent explanation (Gravesmeijer and van Galen, 2003). Van de Walle et al.

(2014) argued that “The most effective learning for a given student occurs when the activities of the classroom lie with in his/her zone of proximal development” (p. 6).

Gravesmeijer and van Galen’s (2003) approach was built upon the platform of number sense. They held that math fact mastery needed to connect to the students’ collection of number relationships. Per their theory, teachers should allow the students to go through the discovery process, rather than dropping the facts onto them. Mathematics was to be viewed as an activity and a time of discovery for students. The key was for students to work at their appropriate developmental level and their own pace. Gravesmeijer and van Galen (2003) dubbed this discovery process guided reinvention. With the teacher’s help, the students could be guided toward more efficient strategies to become fact fluent.

In 2006, Baroody delineated the three developmental stages of math fact acquisition. The stages were as follows: the initial phase involved counting strategies, such as using manipulative materials, fingers, tally marks, or verbal counting. The second phase involved reasoning strategies to determine an answer. This included the use of derived facts, where already known basic facts, composed together, gained the sum or product of an unknown basic fact. The final phase was mastery, or the efficient solving of the problem. Baroody (2006) argued that the first two phases were the essential foundation for conceptual understanding and developing reasoning strategies in order to attain basic fact fluency. He further stated that skipping over the second phase in order to get to mastery was harmful to students’ overall mathematical growth. It “robs students of mathematical proficiency” (p. 27).

In 2010, the Common Core State Standards for Mathematics included the expectation that students needed to reach fluency in their basic multiplication facts in third grade. This version of the standards is still in effect today. The standards assert that fluency is more comprehensive than basic recall. Fluency involves flexibility, accuracy, efficiency and the appropriate use of strategies. Students are also expected to know the facts from memory (CCSSI 2010, p. 23). However, Kling and Bay-Williams (2015) argued that knowing facts from memory was not synonymous with memorization. They stated that recalling facts from memory was obtained through “repeated experiences with the number [so that] students can come to ‘just know’ that 2 x 6 = 12 without ever having had to memorize it” (p. 551).

Research has been conducted to determine how to effectively teach all students their multiplication facts. Woodward (2006) conducted a research study to examine the value of integrating strategic instruction with traditional timed test practice. Using 58 fourth grade students of a wide range of academic abilities, Woodward divided the students into two groups: the comparison group and the intervention group. The two groups were balanced in their ranges of abilities based on their scores on standardized tests. Both groups received 25 minutes of fact instruction daily for 4 weeks. The comparison group was taught in the timed practice skills approach and controlled practice. The intervention group was taught through an integrated approach, where facts were strategically introduced and the practices of derived facts, such as doubling and doubling again, were encouraged for tackling more difficult facts. Both strategic instruction and timed test practice were used to learn their basic facts. The results of the study showed that both groups made progress and were effective in attaining automaticity in basic multiplication facts. However, students in the integrated group scored stronger overall on the post-test and in subsequent maintenance tests that measured the application of basic facts to extended facts and estimation skills.

Similarly, Chung (2004) conducted research on the effect of two different theoretical models, constructivism and traditionalism, for teaching third graders basic multiplication facts.

The study consisted of four 3rd grade classrooms, totaling 71 students. The experiment was a pretest and posttest design with nonequivalent groups. The treatment included 10 math lessons of 30-40 minutes prepared by the researcher with input from the teachers. Two of the classrooms were taught using a constructivist approach: they were given manipulative materials and walked through the concrete, representational and finally abstract phases of the process. Meanwhile, the other two classrooms learned from the school district's traditional math curriculum. While the experiment did not yield significant differences in the students' score, Chung argued that the students were not used to a constructivist style of teaching and so the learning or adjustment curve was high. The constructivist method was also challenging to the teacher and the use of manipulative materials were more challenging to classroom management. The results of Chung's study suggested that a longer period of study using constructivist approaches may be merited.

Zhang, Xin, Harris, and Ding (2013) addressed the difficulty of reaching math fact fluency as well. Three third grade students with math difficulties took part in the teaching experiment. The purpose of the study was to determine the effectiveness of a strategic training (ST) program for students with mathematical difficulties by promoting development of multiplication skills. Zhang et al. (2013) applied the overlapping wave theory (Siegler, 2007) to identify the most effective strategies to help the students progress in their basic fact acquisition problem solving ability. After the baseline assessment was given, students' responses were coded and graded, and then the intervention was planned. The participants received ST intervention, which consisted of progress monitoring and selective task assignment. It also included trial-by-trial instruction which involved students using their own strategies and the teacher providing feedback, asking students to explain why an answer was correct or incorrect, and then the teacher demonstrating multiple strategies to emphasize the next level, or phase, of strategies. Through the use of assessment and individual intervention, all the students progressed in making more effective reasoning choices to solve basic fact problems. Zhang et al. (2013) were able to lead the three students toward more effective choices as the students learned. A combination of explicit modeling, self-exploration and selected task assignment were necessary to provide student growth. The research sample was small; however, by providing explicit instruction through demonstration and providing feedback and allowing student exploration through encouraging use of students' own strategies when needed and self-explanation, the researchers found these to be effective strategies for fact acquisition for students with math difficulties.

To determine the effect of basic fact knowledge to overall mathematical performance in students with math difficulties, a 2016 research study conducted by Nelson, Parker, and Zaslofsky assessed the relative value of basic math fact growth to overall math proficiency. The study involved students in grades four through eight, with a total of 1,493 students, all of whom were part of the Minnesota Math Corps (MMC) intervention program. Half of the students in the study were in fourth and fifth grade. Students were selected to be in MMC because they scored below proficient on the state math test the year before. In the MMC, students received an average of 71 minutes of instruction per week with a total of 50 sessions led by a trained tutor. The students enrolled in MMC took five math fluency tests within the school year. The tests contained all four basic operations: addition, subtraction, multiplication and division. As struggling students increased their math fact fluency, they tended to score stronger on the state test. The researchers suggest the importance of teaching fact fluency and grade-level skills concurrently to those in the upper grades.

In conclusion, strategic instruction, encouraging the complexity of arithmetical thinking, taking time, seeing numbers as relational versus simply numbers, respecting the logic of math as well as the developmental abilities of children are all key aspects of basic multiplication fact acquisition. Working within these premises, all students, even those with mathematical difficulties, can build their mathematical reasoning and become fluent in the basic multiplication facts.

Methods

Participants

The participants were twenty-three third grade students who attend a suburban Christian school in a northern suburb of Minneapolis, Minnesota. The school is made up of middle-class families. Fourteen of the students are girls (60.9%) and nine of the students are boys (39.1%). All of the students are eight or nine years of age. Twenty students (87.0%) are Caucasian, two students (8.7%) are Hispanic and one student (4.3%) is African-American. Twenty-one students (91.3%) speak English as their primary home language and two students (8.7%) speak Russian as their home language. One student has an Individualized Service Plan due to autism and one student has an Individualized Service Plan due to Attention Deficit Hyperactivity Disorder and a Specific Learning Disability.

Materials

The materials for the study was two teacher-generated pretests and posttests and the Measures of Academic Progress (MAP) test which was given in October, 2016, and was administered again in February, 2017. Other materials include manipulative objects, games, timed tests, IXL Math computerized practice and formative assessments.

Design

The study involved twenty-three third grade students. One student transferred in November and had not taken a MAP test. Twenty-two of the students took the MAP test in

October of 2016. The Northwest Evaluation Association (NWEA), producers of the MAP test, delineated the students into five categories: Low (percentile < 21), Low Average (percentile 21-40), Average (percentile 41-60), High Average (percentile 61-80), and High (percentile > 80). Of the twenty-two students, 1 student (4.5%) scored in the Low Average category, 1 student (4.5%) scored in the Average category, 11 students (50%) scored in the High Average category, and 9 students (41%) scored in the High category.

The students were expected to be active manipulators of the materials to assist in developing relational understanding of the 100 basic multiplication facts. Throughout the study, students were welcome to revert back to manipulative objects to help make sense of a concept. The students were given opportunities to explore the principles of multiplication through guided reinvention, that is, constructing their own methods to solve a multiplication problem, describing the process and the teacher (or another learner) providing feedback as to why an answer was correct or incorrect and demonstrated strategies to emphasize progress toward the next level, or phase, of fact development.

Procedure

The design of the research was a correlational study to determine whether the use of constructivist methods, which included strategies such as manipulative objects, guided reinvention, developmental pacing, interactive dialogue and relational understanding, in the teaching of basic math facts would have a significant impact on student understanding of mathematical concepts. The study also determined if there is a significant impact in a specific tier of learners. The independent variable was the implementation of constructivist teaching methods in the introduction of multiplication facts. The dependent variable was the student performance on the MAP test.

To administer the study, the researcher provided on average one hour of math instruction and practice per school day. Students were part of individual, paired, small group and full class learning. When working in pairs, the researcher leveled the games and activities and intentionally paired up students to maximize students' ZPD. The formation of the small groups for guided instruction three times a week was determined through the researcher reviewing daily formative assessment data to maximize students' ZPD. When engaged in the math rotations thrice weekly, one group was with the researcher, another group was working on a daily assignment and flashcards, and the third group was involved in a math activity. For one of the three days each week, the top ten fall MAP math scoring students were part of a math enrichment program that took place outside of the classroom. The math enrichment involved problem solving, but did not include direction instruction of multiplication basic facts.

The material covered in the study was introduced in a specific order. First, the researcher presented the concept of multiplication via repeated addition, equal groups and arrays. Then the foundational facts of 2, 10, 5 and 1 were introduced. Students worked with these facts until mastery, or phrase three, was reached. Next, students worked to use derived facts to eventually reach phase three mastery in the multiples of 3 and 4. Finally the researcher introduced the larger facts of 6, 7, 8, and 9 as derived facts.

Throughout the study students took twice weekly timed tests to check speed of recall. Formative assessments were in the form of observations, dialogue, exit tickets, and daily assignments. The students worked on the online math site, IXL Math, for further practice. The study was 30 days in length.

Once the basic math fact lessons were complete and the posttests and the MAP test had been administered, the data from the MAP tests were studied to find the individual growth of the students. Results from the MAP test were used to discover if any significant gains had been made in the students' test scores and which group of students made the greatest gains - Low Average, Average, High Average or High; and Male or Female.

Results

This study sought to address the following research question: does applying the theory of constructivist learning to the strategic instruction of basic multiplication facts, through the use of strategies that include exploration, manipulative objects, guided reinvention, developmental pacing, interactive dialogue and relational understanding, have a significant impact on student understanding of mathematical reasoning concepts?

Prior to the treatment, thirteen female and nine male third-grade students were given the Fall MAP (Measures of Academic Progress) test on the 25th and 28th days of school. Within the MAP Math test, four strands of mathematical concepts were tested. The four strands were number and operation, algebra, geometry and measurement, and data analysis. The Northwest Evaluation Association (NWEA), producers of the MAP test, assigned a RIT (Rasch Unit) score to the overall achievement and to the achievement in each of the four strands of the MAP Math test. The students' overall Fall MAP Math scores and strand scores are in Table 1.

Table 1

Fall MAP Math Overall Scores

Student	Overall	Number & Operation	Algebra	Geometry & Measurement	Data Analysis
1	201	208	209	202	188
2	198	203	198	186	207
3	202	203	207	196	204
4	198	195	197	200	200
5	197	194	194	194	206
6	203	201	209	198	202
7	180	182	181	175	181
8	198	197	198	192	206
9	195	195	197	189	197
10	208	199	211	210	212
11	211	213	206	214	211
12	206	197	201	210	215
13	211	212	209	211	211
14	198	191	198	206	197
15	197	200	197	195	198
16	210	206	214	214	205
17	203	203	202	208	200
18	209	209	198	215	214
19	190	195	191	186	187
20	195	190	200	197	194
21	198	198	200	195	198
22	197	201	197	194	195

The NWEA delineated the students into five categories: low (percentile < 21), low average (percentile 21-40), average (percentile 41-60), high average (percentile 61-80), and high (percentile > 80). 41% of students were designated in the high category with RIT scores of 202-211, 50% of students were designated in the high average category with RIT scores of 195-201, 5% of students were in the average category with a RIT score of 190, and 5% of students were in the low average category with a RIT score of 180, as shown in Table 2. Tables and 3 and 4 shows the gender breakdown within the five categories.

Table 2

Fall MAP Math Class Report

Low (%ile < 21)		Low Average (%ile 21-40)		Average (%ile 41-60)		High Average (%ile 61-80)		High (%ile > 80)	
count	%	count	%	count	%	count	%	count	%
0	0%	1	4.5%	1	4.5%	11	50%	9	41%
N/A		student’s RIT score of 180		student’s RIT score of 190		students’ RIT scores of 195-201		students’ RIT scores of 202-211	

Table 3

Fall MAP Math Scores for the Nine Males in the Class

Low (%ile < 21)		Low Average (%ile 21-40)		Average (%ile 41-60)		High Average (%ile 61-80)		High (%ile > 80)	
count	%	count	%	count	%	count	%	count	%
0	0%	1	11%	0	0	5	55.5%	3	33.5%
N/A		student’s RIT score of 180		N/A		students’ RIT scores of 195-201		students’ RIT scores of 202-211	

Table 4

Fall MAP Math Scores for the Thirteen Females in the Class

Low (%ile < 21)		Low Average (%ile 21-40)		Average (%ile 41-60)		High Average (%ile 61-80)		High (%ile > 80)	
count	%	count	%	count	%	count	%	count	%
0	0	0	0	1	8%	6	46%	6	46%
N/A		N/A		student’s RIT score of 190		students’ RIT scores of 195-201		students’ RIT scores of 202-211	

After the 30-day treatment of applying constructivist methods to explore basic multiplication facts the students were given the Winter MAP Math test. The students’ overall and strand score results are listed in Table 5. The Winter MAP test was given on the 105th and 106th days of school. The treatment accounted for 37.5% of the days between the first and second tests.

Table 5

Winter MAP Math Overall Scores

Student	Overall	Number & Operation	Algebra	Geometry & Measurement	Data Analysis
1	204	210	205	201	202
2	204	206	212	204	194
3	204	207	202	210	198
4	206	193	214	214	205
5	204	204	205	207	199
6	209	209	211	209	208
7	198	195	207	196	194
8	204	201	210	194	209
9	212	210	213	215	210
10	215	209	218	211	223
11	212	211	228	213	198
12	204	203	204	205	203
13	208	191	208	208	222
14	209	205	208	208	213
15	206	205	208	204	208
16	213	208	218	210	216
17	216	212	223	218	210
18	212	211	213	212	211
19	204	193	204	205	214
20	212	220	212	206	212
21	209	209	215	204	208
22	202	201	203	199	207

Using the delineations assigned by the NWEA, 32% of the students were designated in the high category with RIT (define) scores of 212-216, 63.5% of students were designated in the high average category with RIT scores of 202-209, and 4.5% of students were in the average category with a RIT score of 198, as shown in Table 6.

Table 6

WINTER MAP Class Report

Low (%ile < 21)		Low Average (%ile 21-40)		Average (%ile 41-60)		High Average (%ile 61-80)		High (%ile > 80)	
count	%	count	%	count	%	count	%	count	%
0	0	0	0	1	4.5	14	63.5%	7	32%
N/A		N/A		student's RIT score of 198		students' RIT scores of 202-209		students' RIT scores of 212-216	

As shown in Table 7, one male in average category with a RIT score of 198, four males in the high average category with RIT scores from 202-209, and four males in the high category with RIT scores of 212-216.

Table 7

WINTER MAP Math Scores for the Nine Males in the Class

Low (%ile < 21)		Low Average (%ile 21-40)		Average (%ile 41-60)		High Average (%ile 61-80)		High (%ile > 80)	
count	%	count	%	count	%	count	%	count	%
0	0	0	0	1	9%	4	44.5%	4	44.5%
N/A		N/A		student's RIT score of 198		students' RIT scores of 202-209		students' RIT scores of 212-216	

As shown in Table 8, ten female students scored in the high average category with RIT scores from 204-209, and three female students scored in the high category with RIT scores of 212-216.

Table 8

WINTER MAP Math Scores for the Thirteen Females in the Class

Low (%ile < 21)		Low Average (%ile 21-40)		Average (%ile 41-60)		High Average (%ile 61-80)		High (%ile > 80)	
count	%	count	%	count	%	count	%	count	%
0	0	0	0	0	0	10	77%	3	23%
N/A		N/A		N/A		students' RIT scores of 204-209		students' RIT scores of 212-215	

Observed growth was calculated from the Fall MAP Math test to the WINTER MAP Math test. In the fall, one male student scored in the low average group. This student exceeded the expected growth gain by 9 points, as noted in Table 9. The student scored in the average group in the winter.

Table 9

Observed Growth of Low Average Score

Student	Gender	Overall Fall RIT Score	Overall Winter RIT Score	Observed Growth	Growth Index
9	M	195	212	17	9

In the fall, one female student scored in the average group. This student exceeded the expected growth gain by nine points, as noted in Table 10. The student scored in the high average group in the winter.

Table 10

Observed Growth of Average Score

Student	Gender	Overall Fall RIT Score	Overall Winter RIT Score	Observed Growth	Growth Index
19	F	190	204	14	6

In the fall, five males and six females scored in the high average group. One male and one female student exceeded the expected growth gain by 9 points, two female students exceeded the expected growth gain by 4 points, one female student exceeded the expected growth gains by 2 points and one female student exceeded the expected growth gain by 1 point. One male student met the expected growth gain, one male and one female student did not meet their expected growth gain by an index of -1, one male student did not meet their expected growth gain by an index of -2, and one female student did not meet their expected growth gain by an index of -4. Table 11 shows the observed growth of high average scores, with gender listed.

Table 11

Observed Growth of High Average Scores

Student	Gender	Overall Fall RIT Score	Overall Winter RIT Score	Observed Growth	Growth Index
9	M	195	212	17	9
20	F	195	212	17	9
5	M	197	204	7	0
15	F	197	206	9	2
22	M	197	202	5	-2
2	M	198	204	6	-1
4	F	198	206	8	1
8	F	198	205	6	-1
14	F	198	209	11	4
21	F	198	209	11	4
1	F	201	204	3	-4

In the fall, three male and six female students scored in the 81 or greater percentile, or the high group. When examining these scores one male student exceeded the expected growth gain by an index of 6, one female student met the expected growth gain, one female student did not meet the expected growth gain by an index of -1, two male students did not meet the expected growth gain by an index of -4, one female student did not meet the expected growth gain by an index of -5, one male student did not meet the expected growth gain by an index of -6, one female student did not meet the expected growth gain by an index of -9, and one female student did not meet the expected growth gain by an index of -10. Table 12 shows the observed growth of high scores, with gender listed.

Table 12

Observed Growth of Fall High Scores

Student	Gender	Overall Fall RIT Score	Overall Winter RIT Score	Observed Growth	Growth Index
3	F	202	204	2	-5
6	F	203	209	6	-1
17	M	203	216	13	6
12	F	206	204	-2	-9
10	F	208	215	7	0
18	M	209	212	3	-4
16	F	210	213	3	-4
11	M	211	212	1	-6
13	F	211	208	-3	-10

When examining the strands within the MAP test, the greatest average gain overall was made in the area of algebra, with growth of 10.31, the second greatest gain was in geometry and measurement with 7.04, followed by data analysis with 6.18, and the least average growth was shown in number and operation with 5.50. The median leaders were again algebra with 6.00 observed growth and geometry and measurement with 7.50. However, number and operation is third with a median growth of 6.00 and the least significant median growth is data analysis with 3.50 point growth. Table 13 shows the overall observed average and median growth in the four strands of math.

Table 13

Observed Growth in the Four Strands of Math

Strand	Fall RIT Average	Fall RIT *Median*	Winter RIT Average	Winter RIT *Median*	Observed Growth Average & *Median*
Number & Operation	199.64	*200.50*	205.14	*206.50*	5.50 *6.00*
Algebra	200.64	*199.50*	210.95	*210.50*	10.31 *11.00*
Geometry & Measurement	199.41	*201.00*	206.95	*208.50*	7.04 *7.50*
Data Analysis	201.27	*204.50*	207.45	*208.00*	6.18 *3.50*

Table 14 shows the overall observed average and median growth of the male students in the four strands of math. When examining the male students' scores on the strands within the MAP test, the greatest average gain was made in the area of algebra, with an average growth of 15.66, the second greatest gain was in geometry and measurement with 10.11, followed by number and operation with 7.11, and the least average growth was shown in data analysis with an average observed growth of 3.11. The median leaders were again algebra with 14.00 observed growth as well as geometry and measurement with 14.00. However, data analysis is third with a median growth of 7.00 and the least significant median growth is data number and operation with 5.00 point growth.

Table 14

Observed Growth in the Four Strands of Math - Males

Strand	Fall RIT Average	Fall RIT *Median*	Winter RIT Average	Winter RIT *Median*	Observed Growth Average & *Median*
Number & Operation	199.00	*201.00*	206.11	*206.00*	7.11 *5.00*
Algebra	196.78	*198.00*	212.44	*212.00*	15.66 *14.00*
Geometry & Measuremen	197.89	*194.00*	208.00	*208.00*	10.11 *14.00*
Data Analysis	200.89	*200.00*	204.00	*207.00*	3.11 *7.00*

Table 15 shows the overall observed average and median growth of the female students in the four strands of math. When examining the female students' scores on the strands within the MAP test, the greatest average gain was made in the area of data analysis, with an average growth of 8.31, the second greatest gain was in algebra with 6.61, followed by geometry and measurement with 5.77, and the least average growth was shown in number and operation with an average observed growth of 4.38. The median leader was algebra with growth of 9.00. The second greatest observed gains were in number and operation as well as geometry with 8.00 . Finally, the least significant median growth is data analysis with 6.00 point growth.

Table 15

Observed Growth in the Four Strands of Math - Females

Strand	Fall RIT Average	Fall RIT *Median*	Winter RIT Average	Winter RIT *Median*	Observed Growth Average & *Median*
Number & Operation	200.08	*199.00*	204.46	*207.00*	4.38 *8.00*
Algebra	203.31	*201.00*	209.92	*210.00*	6.61 *9.00*
Geometry & Measurement	200.46	*198.00*	206.23	*206.00*	5.77 *8.00*
Data Analysis	201.54	*202.00*	209.85	*208.00*	8.31 *6.00*

The overall average observed growth of males was 9.00 and the overall median growth was 11.00. The overall average observed growth of females was 6.32 and the overall median growth was 5.00. The results are shown in Table 16.

Table 16

Overall Average and Median Scores of Males and Females with Observed Growth

Gender	Fall MAP Average & *Median*	Winter MAP Average & *Median*	Average & *Median* Observed Growth
Male	198.67 *198.00*	207.67 *209.00*	9.00 *11.00*
Female	201.31 *201.00*	207.54 *206.00*	6.23 *5.00*

Discussion

Overview of the Study

The study sought to determine whether the use of constructivist methods in the teaching of basic multiplication facts would have a significant impact on student understanding of mathematical concepts. The study also sought to determine if there was a significant impact in a specific tier of learners as identified by the NWEA. The independent variable was the implementation of constructivist teaching methods in the introduction of multiplication facts to third grade students. The dependent variable was the observed growth comparison of the student performance from the fall to the winter MAP math test scores.

Summary of Findings

The results of the pretest, treatment, and posttest show there was a correlation between the use of constructivist methods for the teaching of basic multiplication facts and increase of overall understanding of math concepts in some learners. Analysis of the observed growth between pretest and posttest show that the low-average scorer on the fall MAP math test increased achievement by a RIT score growth of 16 points, exceeding the observed growth expectation by nine points, and as of the winter RIT score progressed to the average category. The student who scored in the average range on the fall MAP math test had observed growth of 14 points, exceeding the expectation by 7 points, and in winter progressed to the high average category. Of the 11 students who scored in the high average category on the fall MAP math test, seven of those students met or exceeded growth expectations, and two students moved from high average to the high scoring category. The nine who scored in the high scoring category did not show a positive correlation to the use of constructivist methods for the teaching of basic multiplication facts and increase in overall understanding of math concepts. Only two of the nine students met or exceeded their expected growth.

When viewing the four strands of the MAP math test, i.e., number and operation, algebra, geometry and measurement, and data analysis, there was a correlation between the use of constructivist methods for the teaching of basic multiplication facts and algebraic reasoning. Boys specifically showed an average observed RIT score growth of 15.66 and median growth of 14.00. Expected growth was seven points. The girls' average observed RIT score growth was 6.61 with a median growth of 5.00.

The nine males in the class showed a positive correlation between the use of constructivist methods for the teaching of basic multiplication facts and increase of overall understanding of math concepts. The average observed growth in the RIT scores of males was 9.00 with a median growth of 11.00. The average observed growth in the females' RIT scores was 6.23 with a median growth of 5.00. The expected RIT growth was seven.

Recommendations

Based on the given data, the researcher saw the value of the constructivist methods for the teaching of basic multiplication facts to third grade students. While a 30-day treatment did not produce significant improvement for many of the girls nor for the students who already were high scorers on the MAP test, the support for the low-average to high-average scorers of the class as well as the majority of the males in the class positively gained from constructivist methods.

The researcher saw the value of not giving in to temptation of rushing through the stages of math fact acquisition, but rather offer students ample opportunity to actively interact with the mathematical principles. Particularly for students who are not high scorers on the math MAP tests, the need for exploration and active manipulation of concrete materials and interactive dialogue solidified students' math learning. The students were given the opportunity to explore Baroody's second phase of math acquisition, providing them opportunity to work at their own developmental pace. Guided reinvention allowed students to discover for themselves the patterns embedded in math. As Dornbierer-Schat stated in the Fall 2016 *Voice*, "Math isn't just a process of working through a set of predetermined operations to get the right answer. When taught well, mathematics involves wonder, surprise, even delight" (p. 23). To the researcher, that was the underlying hope of the study of the basic multiplication math facts.

Limitations of the Study

Though the researcher worked thoughtfully to prepare and execute the study, there were limitations involved. The study was limited to a small sample over a short period of time, and the longitudinal effects of the study are not known. The class was made up of a fairly homogenous group of 23 students. Ideally, this testing method would be tried in a wide variety of heterogeneous classrooms over a longer period of time.

The time between the pretest and the posttest included more than the treatment's instruction. The researcher worked consistently to incorporate constructivist principles, or meaning-making, throughout the time between the tests, so this treatment's flow did not differ greatly from the presentation of other materials in the math classroom. However, more than basic multiplication fact acquisition was taught between the tests.

Another limitation was noted in the MAP test findings. According to NWEA, students who had an observed growth of $\pm$ 5 or wider, the observed growth standard error could be large enough to put the outcome in question. In that case, other collection data should be considered. Nine students in the class met this criteria.

When considering future research for the topic of basic multiplication fact acquisition for elementary students, there are other areas of research to consider. For example, how to help high scoring students strengthen and further their math understanding. Some of the students may have been held back as the rest of the class worked toward fluency on the basic facts. What kind of learning enrichment should occur that dovetails with the basic multiplication teaching? Would it have helped for the ten high scoring students to stay in the classroom, with enrichment coming to them, rather than be pulled out once a week?

The need for number sense came up often in research as a pre-requisite for basic multiplication math fact acquisition. How can the younger grades build that strong foundation for multiplication readiness?

Furthermore, questions arise about the methods of testing. In what ways do timed tests hinder and/or help the students who process at a more methodical rate? Does student choice of type of timed test help in the acquisition of basic multiplication facts? What are quality methods in lieu of timed tests to track fluency? What other pretests and posttests are available to accurately gauge impact of constructivist principles on the teaching of basic math multiplication facts and its impact on overall math reasoning growth?

Overall, more study would be helpful in the area of teaching for meaning making while improving speed of recall on basic math multiplication facts.

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