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Sourcebook in the Mathematics of Medieval Europe and North Africa (Book Review)

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Abstract
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REVIEWS

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Sourcebook in the Mathematics of Medieval Europe and North Africa, Victor J. Katz, Menso Folkerts, Barnabas Hughes, Roi Wagner, and J. Lennart Berggren, Eds., Princeton University Press, Princeton, 2016. xvi+574 pp., ISBN 978-0691156859. $95.

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> Mathematics is a living plant which has flourished and languished with the rise and fall of civilizations. Created in some prehistoric period it struggled for existence through centuries of prehistory and further centuries of recorded history. It finally secured a firm grip on life in the highly congenial soil of Greece and waxed strong for a brief period. In this period it produced one perfect flower, Euclidean geometry. The buds of other flowers opened slightly . . . but these flowers withered with the decline of Greek civilization, and the plant remained dormant for one thousand years.
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> Such was the state of mathematics when the plant was transported to Europe proper and once more imbedded in fertile soil. By A.D. 1600 it had regained the vigor it had possessed at the very height of the Greek period and was prepared to break forth with unprecedented brilliance.
>
> Morris Kline [**10**, pp. 10–11]

I first fell in love with the history of mathematics as an undergraduate. I had enjoyed mathematics ever since learning how letters could be used to solve story problems, but in college I also became intrigued by the history of philosophy, particularly by how ideas from an earlier period could strongly influence later thinking. So when I discovered Morris Kline's *Mathematics in Western Culture* [**10**], a popular exposition of the cultural significance of mathematics through the ages, I was hooked. I glimpsed a way to combine my academic interests by focusing on the history of mathematics.

I suspect Kline's book, like E. T. Bell's earlier *Men of Mathematics* [**1**], first published in 1937, has given pleasure to many of us over the years. While both authors are good storytellers, a good story does not always convey accurate history. I am not just referring to factual misrepresentation or to the use of oversimplified storylines and conjectural filler. More substantively, I have in mind the ways we conceptualize the overall project—what content and methods we consider to be genuine mathematics and how we see various cultures contributing to the enterprise. As I was inducted into the discipline of the history of mathematics in the late 1970s, I learned that the Eurocentric outlook promoted by Kline and others needed serious modification. This is so even if one is primarily interested in the history of mathematics as the heritage behind what we presently study in our mathematics classrooms.

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Unlike mathematics itself, history of mathematics is a relatively young discipline.[1] Professional interest in the history of mathematics intensified toward the end of the nineteenth century, when scholarly editions of Euclid, Archimedes, Diophantus, and Apollonius appeared. Popular English-language histories of mathematics were published shortly thereafter. These were intended to be used in educational settings and covered mathematics from Greek times into eighteenth- and nineteenth-century Europe, with brief attention given to non-Western cultures and to the so-called "dark ages" of the medieval era.

At this time, earlier mathematical ideas originating in Egyptian and Babylonian cultures were treated as primitive beginnings. The mathematics of these cultures, along with those of China and India, were measured by the standards of later theoretical mathematics and found wanting. They were judged as coming up far short of the ancient Greeks' accomplishments.

India was credited with producing our numeration system and simple algebra, though some suspected that the latter was based on earlier Greek ideas and methods. The medieval period in Europe prior to 1200 was pronounced barren and devoid of originality. Arabic scholars' involvement during the same time was seen primarily as one of preservation, translation, and transmission, passing on the mathematics inherited from India and Greece to the Europeans who came after them. Historians working at this time judged this role significant, however. Once European scholars discovered the intellectual treasure possessed by Arabic culture, they began to translate Greek works into Latin. This was thought to set in motion the eventual rebirth of rationality and a true understanding of the world, mathematics being both the pinnacle and necessary foundation for such a renaissance.

My characterization of early accounts of the history of mathematics needs some refinement to be completely fair, but it is largely the story put forward a century or so ago. It clearly underlies Kline's mid-century work quoted at the outset, and it still has a more popular following than it deserves. The Eurocentric bias of this story fit with what was then thought about the nature and development of mathematics. The nineteenth and early twentieth centuries had witnessed significant shifts toward mathematics becoming more theoretically organized and logically grounded—that is, toward what many believed it was truly meant to be, the origin of which was plainly due to the Greeks.

Resources for challenging this outlook started to appear in the 1920s and 1930s. Scholarly editions of *The Rhind Mathematical Papyrus* were published, and the vast extant collection of mathematical cuneiform tablets was analyzed by Otto Neugebauer and others. This led to a much richer understanding of ancient Egyptian and Mesopotamian mathematics and their connections to Greek mathematics [**11, 16**]. Babylonian mathematics around 1800 B.C., in particular, was elevated in importance by discovering that scribes had developed clever ways to solve quadratic and some higher-degree equations, apparently through an abstract algorithmic approach. This progress notwithstanding, Smith's two-volume *A Source Book in Mathematics* [**15**], published in 1929, written to supply teachers and students of mathematics with "excerpts from the works of the makers of the subject," still only contained selections from European books printed between 1478 and 1900. Cajori's grand two-volume *History of Mathematical Notations* [**3**], published in 1928 and 1929, included a number of items drawn from non-Greek cultures (Babylonian, Egyptian, Indian, Chinese,

[1] The anthology edited by Dauben and Scriba [**4**] documents the global development of historical writing on mathematics from ancient times through the twentieth century.

Arabic) as well as from the medieval European period, but the bulk of these books focused on Greek and European mathematics.

New information about Egyptian and Babylonian mathematics helped fill in details about the early history of mathematics, but the overall storyline remained the same. The Greeks, it was thought, might have obtained some primitive knowledge of geometry from the Egyptians, but they went far beyond it in their own research, and the Greeks seemed to justify the algebraic procedures of the Babylonians by transforming them into a sort of geometric algebra. Moreover, the Greeks' logical organization of mathematics and emphasis on proving universally valid results from self-evident axioms were presented as their own ingenious contribution. Non-Greek contributions were still, at best, preparatory to the real thing.

History of mathematics became a more professional enterprise after World War II. As a premier branch of the history of science, research articles on the history of mathematics often appeared in scholarly journals such as *Isis* or the newer *Archive for History of Exact Sciences*.

Textbooks in history of mathematics began to be updated during this time period. Such books were written both by mathematicians-turned-historians and by trained historians of mathematics. Howard Eves's *Introduction to the History of Mathematics* [**5**], published in 1953, was of the former variety. It found wide acceptance from the start. Its popularity in mathematical circles was due in part to its inclusion of multipart "problem studies" for students to explore using their own knowledge of mathematics. Carl Boyer's *A History of Mathematics* [**2**], published in 1968, was of the second variety and was partially based on his own historical research. Exercises in Boyer's text were more historically focused than those in Eves's text. They included essay questions on historical connections and significance as well as mathematical problems that students had to solve using methods available from the chapter's time period.

History of mathematics came of age in the early 1970s with the founding of the International Commission on the History of Mathematics and its official journal, *Historia Mathematica*, both of which still exist. Since that time, history of mathematics has seen increased institutional and professional support, and the number of career historians of mathematics has grown. In addition to producing an expanding collection of secondary material, historians have compiled several new anthologies of primary source material in translation. Sourcebooks published in the 1960s and 1970s fleshed out our understanding of several aspects of classical European mathematics between 1200 and 1900.

During this time, historians of mathematics continued to see education as a prime area where their expertise could be beneficial. The International Study Group on the Relations between History and Pedagogy of Mathematics dates back to the early 1970s. Its Americas Section has been active since 1984. The Mathematical Association of America's (MAA) special interest group HOMSIGMAA, founded in 2001, is thriving, and workshops and sessions devoted to history of mathematics are well attended at mathematics conferences. The MAA's various book series include around forty books on historical topics, and their online journal *Convergence* has provided historical resources for college mathematics classrooms for over a decade. The American Mathematical Society's (AMS) History of Mathematics series contains another forty-five books in this area.[2]

[2]The website for the MAA's book series is http://www.maa.org/publications/books/book-series. Their online journal *Convergence* can be found at http://www.maa.org/press/periodicals/convergence. The AMS book series in the history of mathematics can be found at http://bookstore.ams.org/HMATH.

Concomitant with the increase in historical research and writing for specialists and educators over the last few generations, there has been a fundamental reassessment of historical perspectives. The earlier Eurocentric outlook is now strongly contested, and the work of other cultures has been given its due as genuine parts of mathematics' global development [**8, 14**]. This trend has not occurred in isolation, of course; our world has become a global village in the interim, and multiculturalism has found a home in many academic disciplines. At the same time, mathematics is no longer universally viewed as a collection of eternal truths about abstract entities linked together by a web of logic. The philosophy of mathematics now explores aspects of mathematical practice that go beyond polished definitions and theorems.

Undergirding and supporting these perspectival changes is an ever-growing awareness of other cultures' accomplishments. Articles, monographs, and sourcebooks have expanded into new areas, presenting and exploring primary source material from medieval European, Egyptian, Mesopotamian, Chinese, Indian, and Islamic countries, among others. This work has transformed our understanding of the contributions of these cultures to mathematical knowledge. Some of this work, such as Høyrup's groundbreaking work on the dynamic geometrical substrate of Babylonian algebra [**6**], has given us a completely new interpretation of the mathematics itself. Other works have positioned mathematics within the broader socioeconomic context of the various cultures [**7, 12, 13**].

Ten years ago, the time was ripe for consolidating the scholarship on the historical development of mathematics in several major non-Western cultures. Under the general editorship of Victor Katz, the 675-page sourcebook *The Mathematics of Egypt, Mesopotamia, China, India, and Islam* [**9**] was published, making readily available in English translation a wealth of primary source material. The book's introduction made a thoughtful case for taking a more culturally inclusive view of the history of mathematics. The chapters included long selections along with some contextual commentary on a variety of mathematical topics.

This brings us to the 2016 work under review here. This book serves as a companion volume to the 2007 sourcebook just mentioned. Given the limited size of that work, many developments in the under-represented medieval period had to be excluded. These are now treated by this sourcebook in an additional 590 pages. Together, these two sourcebooks superbly document the scope and depth of non-Greek mathematics produced prior to the modern era.

As the title indicates, the geography under consideration is medieval Europe and North Africa. The time period covered is from roughly 800 to 1500. Three long chapters are devoted to roughly 110 substantial mathematical excerpts taken from Latin, Hebrew, and Arabic writings. Half of these pieces are newly translated from original sources; the rest have been gathered from numerous disparate publications. Some of these are also translated into English for the first time. Selected and edited by experts in their field, this book offers a wealth of new primary source material.

The material in the first chapter, on Latin medieval mathematics, is best known. It consists of writings associated with early Latin schools, medieval universities, and abbacus schools. There are passages from Boethius, Gerbert, Fibonacci, Jordanus, Oresme, Regiomontanus, and others. Topics covered include arithmetic, recreational mathematics, geometry, algebra, kinematics, and trigonometry. People acquainted with earlier accounts of this period will have little difficulty navigating this chapter. The selections here provide a welcome compilation into one place of primary source material not only for familiar developments, but for some off-beat and less well-known episodes as well.

The study of medieval Hebrew mathematics, the focus of chapter two, is relatively new. Up to now, this material has been less accessible than those for medieval Latin mathematics. Not much has made its way into standard history of mathematics textbooks. Almost 160 pages of fairly lengthy selections, from various locales, are given here. The polymath Levi ben Gershon, who wrote religious, philosophical, and scientific works in early fourteenth-century France, composed the most original mathematics in this tradition and is well represented. His theoretical arithmetic, containing theorems and proofs, reminds one of Jordanus's work from the previous century. His work also showed readers how to calculate combinations and permutations and provided inductive proofs for his results.

The earlier sourcebook contained 160 pages on mathematics from Medieval Islam. That chapter contained commentary about and extracts from better known mathematicians living in eastern Islamic countries, excerpting the work of mathematicians such as al-Khwārizmī, Omar Khayyām, and others. This sourcebook has a slightly larger section of almost completely new selections written by scholars living in the western Islamic regions of Spain and North Africa. Even readers familiar with the history of mathematics will most likely not know many of these authors or excerpts. Many passages discuss standard topics: computational operations on positive whole numbers and fractions, combinatorics, geometry (practical and theoretical), algebra, and trigonometry. Such selections are clearly related to writings in other Islamic countries at the time, and the mathematics developed here was evidently familiar to some later medieval Latin mathematicians. Leonardo Fibonacci, for instance, learned his mathematics as a child living in North Africa. Some of the types of problems, treatments, and notation documented in this chapter can be seen in his *Liber Abbaci*. However, the mathematics excerpted here occasionally highlights something more original. From ibn Hūd's eleventh-century survey of the mathematics of his time we have, among other things, a new proof of Heron's theorem as well as the statement and proof of Ceva's theorem, named after an Italian mathematician who lived six hundred years later.

Like the other two chapters, this last one has a general introduction, section introductions, and commentaries that contextualize the various selections and point out connections between mathematicians and cultures. However, because of the unfamiliarity of the material in this (and the second) chapter, I found that I invariably wanted more than was supplied. Even simple things like providing more information about specific dates and places would have helped; a map would have been useful in addition to the author timeline given in Appendix 4. This would be especially welcome on account of the editors' decision to present the material topically within each chapter, beginning from the most elementary to the most advanced. Thus, even within a topic the selections do not always proceed chronologically or by author. However, given that the study of the history of Hebrew mathematics and the western branch of Islamic mathematics is still in its infancy, one ought not complain too vigorously about not having a more definitive narrative connecting the selections. This book at least gives us some primary source material and commentary for starting to develop such an account.

Sourcebook in the Mathematics of Medieval Europe and North Africa will probably not find a very broad audience among mathematicians. It should certainly be available in libraries, though, as a resource for faculty and students. Its primary use will undoubtedly be as supplementary source material for courses in the history of mathematics, but education courses for elementary and secondary school mathematics teachers might profit from it as well. Few will read it cover to cover, but students (and their teachers) may find it interesting and instructive to learn how mathematicians in earlier cultures treated the sorts of material we study today. Hopefully, some will find the material attractive enough to continue the sort of scholarly research begun here, further extending our understanding of mathematics as an international endeavor.

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