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Making and Breaking Mathematical Sense (Book Review)

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Abstract
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Making and Breaking Mathematical Sense

A popular philosophical problem mathematics students sometimes puzzle over is whether mathematics is discovered or invented. These options are not binary opposites, of course, and answers may vary depending on which parts of mathematics are considered and how the terms are defined. Classical foundational programs take different sides on this, but in doing so they all focus on the reality of the objects, properties, and relations studied in mathematics, on the truth character of mathematical statements made about them, and on how we humans can come to know these things. Such ontological and epistemological issues, according to Roi Wagner in *Making and Breaking Mathematical Sense: Histories and Philosophies of Mathematical Practice*, are both too narrow and too static; they should not be our central concern as we reflect on the nature and practice of mathematics.

Wagner, like a number of others over the past few decades, is far more interested in how mathematics is actually produced, accepted, and applied (its social construction) than in how it can be conceptualized as a finished body of knowledge (its essential nature). In particular, he is keenly committed to exploring the shifting meaning of mathematical textual material as it undergoes development and transformation by human actors embedded in different cultural and intellectual settings. Wagner intentionally adopts a post-structural stance on the practice of mathematics: what is important is not some stable theoretically organized formal outcome, but the ways in which mathematical notions and notations obtain and change sense (semiosis) as they get formulated, used, and communicated. “Meaning,” he says elsewhere (in *Post-Structural Readings of Gödel’s Proof*), “is an operator which produces difference across repetition.”

This sort of postmodern perspective first came to prominence in literary circles, but it is now (and here) also making inroads in contemporary philosophy of mathematics. Readers unfamiliar with (or antagonistic to) this polemical point of view and its often perplexing dialect may be put off by some parts of this book, but Wagner makes a concerted effort to limit technical jargon and to restrict denser philosophical discussions to particular chapters that can be skimmed or omitted without losing the overall gist of his argument. In fact, Wagner mostly writes with a clarity and an engaging style that pulls his reader along as he makes a case for his maverick approach.

As the book’s subtitle indicates, this work treats both history and philosophy of mathematics. Ambitious as this goal is, it is more as well, because Wagner develops his philosophical position partly through dialogue with contemporary theories of cognition. The main thrust of his argument, however — that mathematics is replete with shifting interconnected meanings and ambiguities that aren’t adequately captured by the formalized systems that abstract from them — is amply illustrated via well-documented historical case studies.

Wagner begins by critically testing classical philosophies of mathematics (empiricism, Platonic realism, logicism, intuitionism, formalism, logical positivism) vis-à-vis certain issues arising from mathematical practice. Standard foundational approaches, for instance, take little notice of the different ways in which mathematicians have responded to “monsters” (e.g., incommensurables, infinitesimals) that challenged established ways of doing mathematics; such philosophies presuppose a polished product where these have already been barred or tamed. Rather than choose from among the various foundational approaches, Wagner sees them as fixating on aspects of mathematical practice that tell only part of the story.

His alternative vision is for a "constraints-based philosophy of mathematics: instead of debating the reality of mathematical entities, we will think of mathematics as a field of knowledge that negotiates various kinds of real constraints." These include various social constraints but also methodological constraints that promote consensus, such as the formalizability of results and validity of proofs, along with more intangible controls such as perceived relevance and depth. Such a philosophy acknowledges and celebrates the interpretative fluidity and richness of meaning witnessed in mathematical practice. It thus aligns mathematics in certain ways with the humanities and social sciences as well as with natural science and analytic philosophy.

Wagner devotes two of his seven chapters to analyzing episodes in the history of mathematics. Chapter 2 explores some fascinating recent work by Jens Høyrup, Albrecht Heefer, and Wagner himself on two significant advances in Renaissance algebra: how the notion of and notation for algebraic unknowns and variables arose from various mercantile practices involving quantity purchases and currency exchanges; and how the idea of negative numbers and their complex roots gradually evolved from the formal arithmetic of binomials involving subtracted quantities and the notion of debts. These developments demonstrate the use of multiple and shifting meanings, moving both from one domain to another (business transactions to mathematics) and within a given domain (algebra).

Chapter 4 continues to expound on the theme of semiosis, this time through contemporary examples of mathematical practice. His first case study considers the various senses of the "paradigmatic mathematical sign, *x*," as used in developing the theory of power series: as a variable, a transcendental constant, and a placeholder. The role taken by *x* depends on the particular operations and uses being explored for power series, something that often perplexes students and that is not typically clarified by textbooks. The second case study looks at the *Marriage Theorem*. Wagner notes that certain developments here during the last half of the 20th century have some interesting connections to gender role stereotypes. His point here is not to assert that the way we use language dictates our mathematical results, but he does say that "my goal is to make room for the possibility that language co-constitutes scientific (specifically, mathematical) production."

Chapter 5 is devoted to discussing contemporary theories of cognition, both "theories of elementary number processing" and "more complex theories of mathematical concept formation," which involve mathematical metaphors. This is too complex to summarize briefly. Suffice it to say that Wagner identifies difficulties with both sorts of theories, offering instead a post-structuralist approach to mathematical cognition by Deleuze — something Wagner says can be skipped by readers allergic to postmodern styles of reasoning.

He carries the theme of mathematical metaphor forward in Chapter 6, however, when he delves into two more historical case studies, this time used as test cases for theories of cognition. The first of these examines different ways in which the two conceptual domains of geometry and algebra are intertwined in four medieval, Renaissance, and early modern works. The transfer of knowledge that occurred there between these two domains was varied and complex, once again involving entities with plural shifting meanings. His second case study looks at different concepts of infinity and infinitesimals, largely countering features of their historical development to a particular theory of cognition.

The concluding chapter provides a counterpoint to his earlier chapter on philosophy of mathematics. That one discussed ways in which mathematics develops under various constraints; this one investigates "how mathematics changes the reality in which it evolves, feeding back into the constraints that shape it." Wagner's views here are formed in conversation with the post-Kantian German philosophers Fichte, Schelling, and Hermann Cohen — "seen today as obscure oddities" rather than as prominent philosophers one might expect to turn to for insights on mathematics. Wagner values them, however, for what he can learn from their attempts to coordinate empirical science with human thinking, ways that differ from current analytical approaches to philosophy of mathematics, which seem to him to be at an impasse. Inspired by them, Wagner argues that mathematical ideas are designed to fit reality in a general way, but that they also shape our interaction with the world around us by being embedded in our technology and by providing intellectual tools needed for us to make scientific sense of our experience. This being so, it is not all that surprising that mathematics is applicable to the empirical world we experience, which provides Wagner with at least a partial response to Wigner's well-known query about the "unreasonable effectiveness of mathematics in the natural sciences."

This is a book that tackles a number of big issues on several fronts. Not everyone will agree with its point of view, but it nevertheless offers a substantial and interesting treatment of issues in the philosophy of mathematical practice. It does this in part via well-reasoned case studies in the history of mathematics. This combination of history and philosophy of mathematics addresses a pithy criticism about science, which I will paraphrase here: "history of mathematics without philosophy of mathematics is blind, while philosophy of mathematics without history of mathematics is lame." Readers will have to judge for themselves how well this book sees and walks. I, for one, very much appreciated the interactive exposition. This book expands the discourse on philosophy of mathematical practice using informed specifics from history of mathematics, making it a worthwhile addition for academic libraries and historians and philosophers of mathematics.