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Boole's Algebra Isn't Boolean Algebra (Article Review)

Abstract

Reviewed Title: *Boole's Algebra Isn't Boolean Algebra* by Theodore Hailperin. *Mathematics Magazine*, v. 54:4, pp. 172-184. ISSN: 0025-570X

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Boole's algebra isn't Boolean algebra.

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This article, like its parent work [*Boole's logic and probability*, see pp. 87–112, North-Holland, Amsterdam, 1976; MR0444391], is an example of the use of contemporary mathematics to elucidate a classic text in the history of mathematics. Its goals are: (1) to unravel “the nascent abstract algebra ideas” underlying Boole’s approach to logic, and (2) to provide a contemporary algebraic justification of some ideas and procedures that have seemed mysterious or unfounded to Boole’s successors.

The paper begins with some insightful observations on the state of algebra in Boole’s day as compared with later developments. Noting that Boolean algebra is of much later vintage than Boole’s work, the paper goes on to the stronger suggestion that the underlying ideas for Boole’s work are not those of a Boolean algebra of sets but rather those of a more comprehensive structure, an algebra of “signed multisets” which satisfies the conditions of “a commutative ring with unity having no additive or multiplicative nilpotents”. The set of idempotents in such an algebra can be used to construct a Boolean algebra by appropriately modifying the addition operation, but the author observes that Boole did not notice this fact even though Boole made extensive use of the set of idempotents of the larger algebra.

The paper then moves on to the second item, namely, providing a rationale for some parts of Boole’s method which later logicians rejected. These involve Boole’s “uninterpretable expressions”, such as x/y , and his procedure for expanding such expressions to transform them into interpretable forms. By a variation on the techniques of passing to a factor ring and forming a ring of quotients, the author proposes “to reproduce with due mathematical rigor, Boole’s solution of a Boolean equation by division and expansion, and to justify his interpretation for the algebraic solution”.

The exposition exemplifies a much higher level of logical rigor than commonly found in historical essays. In particular, it makes, and for the most part observes, the distinction between algebraic calculi (e.g., Boolean algebra) and algebraic structures (e.g., a Boolean algebra). The corresponding distinction between a formula of a calculus and a property of structures is not as uniformly observed, and Boole’s vague idea of an expression (sic) satisfying a formula is used without clarification. Nevertheless, the level of rigor employed is sufficient to make evident not only that neither a calculus nor a class of structures was defined by Boole but also that discovery of a calculus and class of structures faithful to Boole’s statements is a demanding historico-mathematical project.

Today we consider algebra to involve in an essential way study of algebraic structures, and we consider logic to involve in an essential way study of logical deductions (including mathematical proofs). The author emphasizes the fact that Boole thought of himself as using mathematical ideas to deal with logic. But it is also true, as can be inferred from this article, that Boole had no thought of algebra as a study of structures. There is some irony, therefore, in the fact that this article treats Boole’s work entirely from the perspective of algebra omitting altogether mention of any contributions to the study of deductions that Boole may have made.

The author’s overall approach to Boole’s work is not narrowly historical, but at the same time it is not in conflict with a strict historical approach. Moreover, because of the author’s command of relevant historical material and his efforts to represent the technical aspects of Boole’s work faithfully, historians of algebra and logic can read this article with profit.

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