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Mathematician's Apology

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Abstract
"Better standards may help all math teachers to shape our classes to better reflect the creativity and play at the heart of mathematics."

Posting about ways to improve math's negative image from *In All Things* - an online hub committed to the claim that the life, death, and resurrection of Jesus Christ has implications for the entire world.

http://inallthings.org/a-mathematicians-apology/

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A Mathematician’s Apology

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> *When I meet someone for the first time the conversation usually goes something like this. “So what do you do?” “I’m a professor.” “Yeah, what do you teach?” “Mathematics.” “Oh, I hate math.” “Sorry…”*
>
> *What makes so many people dislike mathematics? To me mathematics is a beautiful subject that has contributed so much to our modern world. I see a disconnect between what mathematicians know and love and what everyday people experience in school. Perhaps what is called “math” in many classrooms is not really a good representation of mathematics. For that I am sorry. As a former high school teacher, I know that I did not always teach math in a way commensurate with my vision of what mathematics really is.*
>
> *To a mathematician, mathematics is about solving problems. Mathematics is not just a collection of “facts” about the numerical and spatial aspects of the world, it is a process by which we use and develop them. The interesting part is in the reasoning, the justification, and the way different concepts are brought together to answer a question.*
>
> *In that way, math is like Legos. In the film The Lego Movie, the villain wants to build a world that never changes. The heroes, in contrast, interact with this world through creativity and play. Both sides are familiar with the same Lego bricks, but they use them to different ends. So it is with mathematics. Too often mathematics is thought of merely as a pre-built ivory tower and the curriculum is the instruction booklet: what one needs to learn, in math class, is how to follow the instructions properly so you can put the “bricks” (pieces of mathematical knowledge) together in the “right” way so as to end up with the “right” result.*
>
> *But mathematics is about seeing what you can build, not just following the instructions. Given this, the role of a good math teacher is three-fold. First to expose the students to new “bricks;” second to teach the students how different bricks fit together; and third to give the students rich tasks in which they can play with the bricks to build new structures.*
>
> *Because of how math is often taught now, students get the perception that mathematics is just a list of facts to be memorized, starting with addition and multiplication tables and perhaps culminating with the quadratic formula sometime in high school. They likely will memorize these facts, maybe to the tune of a song, reinforcing the myth that mathematics isn’t about understanding and applying anything—that it’s not about “playing”—but about being able to reproduce arbitrary combinations of symbols. Given this, it’s no wonder they hate math. They should. (Mathematics is also akin to poetry in a way. Having a rich vocabulary is important to writing good poetry, but it’s the way the words are put together that makes a poem. If you want someone to write good poetry, you have to give them lots of opportunities to play with words, not just memorize their definitions.)*
>
> *So why is math taught so sterilely in school? Perhaps part of the reason is because teachers are under the gun of getting students to perform on high stakes tests. If teaching the students test-taking tricks is more effective at generating good scores on standardized*

> *tests than actually teaching mathematics, then there is a huge amount of pressure to do so. I know I felt that pressure when I was a high school teacher and often catered to the test as well.*
>
> *This emphasis on high stakes testing becomes reflected in the state standards that guide math education. The tests are based on state standards—and those standards are often written to be easily assessed in a standardized test. This circular logic poses big problems for math, because testing for deep conceptual understanding is difficult to do on a bubble sheet. As a result, you end up with state standards like this 8th grade math standard from Nebraska:*
>
>> *8.1.3.e Solve problems involving ratios and proportions (e.g.,)*
>
> *This standard basically tells you what will be on the test and so students are taught to recognize this and apply the "butterfly method" to solve it (I wish I were joking). The unfortunate result is that instead of a mathematics curriculum centered on problem solving and sense making, the students are taught how to solve 35 types of "problems" likely to appear on exams using 35 disconnected methods. The standard isn't written in a way that encourages teachers to engage in problem solving or conceptual understanding. A student can correctly answer the question without really knowing what they are doing or why they would ever need to. But this is a disservice to the students because proportional reasoning is actually incredibly useful.*
>
> *While bad standards can be damaging, good standards can help. As a mathematician, I find some hope that the new Common Core State Standards (CCSS) will help improve the exams, which may help to reinforce the importance of teaching genuine mathematics in schools. This is partly because the three principle authors of the CCSS-Math included a mathematician who worked to include real mathematics in the standards. The key idea is that the process of doing mathematics is written in as standards to be tested.*[1]
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> *For example, and in contrast to the standard above, in the CCSS proportional reasoning is actually seven standards (not one) that outline that math should be connected to problems not just mechanics. Here's one example:*
>
>> *6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams or equations.*
>
> *Certainly the above equation, , fits within the standard, but the standard itself encourages reasoning, understanding and problem solving.*
>
> *Here is a task that is more in the spirit to the CCSS version of the standard:*
>
>> *John's dad has just finished putting together his above-ground swimming pool in the backyard and is now filling it up with the garden hose. It is now 10:00 AM. Will the pool be full for John's birthday party at 4:00 PM?*

> *Notice that there is very little information given. The task requires the student to ask questions in order to solve the problem. How big is the pool? What shape is it? How quickly is water coming out of the hose? The key difference is that the thinking and reasoning is left to the students. Most solutions of this problem will require proportional reasoning at the end, but there are multiple valid ways of getting there. Students will need to justify their reasoning and creativity is encouraged.*
>
> *Students who grow up solving problems, thinking critically and asking questions in school may go on to answer some of the deep unanswered questions that have hounded mathematicians for hundreds of years. For example, are there any integers x, y, and z such that ? See if you can find a triplet that solves the equation…actually don't, because this problem, known as Fermat's Last Theorem, is impossible. In fact there are no positive integer solutions to this problem for any positive integer exponent , i.e. , except when and there are infinitely many solutions to it. This problem lay unsolved from 1670 until 1995 when Andrew Wiles after years of hard work finally provided a proof that no solutions exist. Problems like this, while not on the surface "practical", demonstrate a deep beauty and structure within mathematics reminiscent of art.*
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> *I wonder if more students had math classes where they were given interesting problems and had to work together to solve them using their creativity and critical thinking, I might have to apologize less often. The sterile perception of mathematics is not limited to experiences in school mathematics. Unfortunately, mathematicians and mathematics educators do not always capture the beauty of mathematics in our teaching at the college level either. This means that many teachers are sent out into classrooms thinking that math is just a canon of formulas to memorize. The pressure is on mathematicians to teach our own subject better in the future.*
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> *Better standards may help all math teachers to shape our classes to better reflect the creativity and play at the heart of mathematics–-the creativity and play that triumphs at the end of The Lego Movie. Perhaps someday I will not have to apologize to every new person I meet for liking math. At a minimum, I hope for a world in which the standardized tests we subject our students to actually point toward the kind of mathematics I've described here. Perhaps we might even dare to reallocate the time and money used for these tests to give math teachers the resources to actually teach mathematics. There are already many great teachers doing mathematics justice in the classroom, but I'd like this kind of teaching to be more widespread. Most likely we will need more than superglue to fix the inaccurate perception so many people have of mathematics.*

Footnotes

1. These standards called the Standards for Mathematical Practice get at what it really means to do mathematics. The content standards are in some sense the playground in which students can do the mathematics and the practice standards outline what it means to do mathematics. The National Council of Teachers of Mathematics described this in "Principles and Standards" already in 2000. The CCSS Standard for Mathematical Practice are in line with what was laid out by NCTM, but the genius was that they were written in as standards to be assessed. This way they could steer

assessment back on course and hopefully drive instruction back to real mathematics. The unfortunate byproduct of assessment is that it often shifts the goal from deep learning to easily measurable outcomes. ↩