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Effect of Geometer's Sketchpad on Student Knowledge and Attitude

David J. Dekker

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Abstract

This study examines the effects of Geometer's Sketchpad on student achievement and attitudes in algebra classrooms. It has a quasi-experimental design. The sample included 93 students enrolled in Advanced Algebra courses at Calvin Christian High School in Grandville, MI. Four classes participated in a five-day unit that explored graphs of equalities and inequalities. Two classes used traditional graphing calculators and two classes used Geometer's Sketchpad. One class of each type took a pretest to measure prior knowledge. All classes took a posttest and attitude survey at the end of the study. A t-test was used to compare class averages. There were no significant differences when looking at posttest and attitude survey results. There were significant differences when looking at difference scores from the pretest and posttest. This is an important result as it has implications on the use of technology in the mathematics classroom and in the formation of future studies like this one.

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Effect of Geometer's Sketchpad on Student Knowledge and Attitude

by

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B.A. Calvin College, 2004

Action Research Report
Submitted in Partial Fulfillment of the Requirements for the
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Abstract

This study examines the effects of Geometer's Sketchpad on student achievement and attitudes in algebra classrooms. It has a quasi-experimental design**.** The sample included 93 students enrolled in Advanced Algebra courses at Calvin Christian High School in Grandville, MI. Four classes participated in a five-day unit that explored graphs of equalities and inequalities. Two classes used traditional graphing calculators and two classes used Geometer's Sketchpad. One class of each type took a pretest to measure prior knowledge. All classes took a posttest and attitude survey at the end of the study. A t-test was used to compare class averages. There were no significant differences when looking at posttest and attitude survey results. There were significant differences when looking at difference scores from the pretest and posttest. This is an important result as it has implications on the use of technology in the mathematics classroom and in the formation of future studies like this one.

In 2000, the National Council of Teachers of Mathematics (NCTM) released its *Principles and Standards for School Mathematics*, which lays out goals and expectations of mathematics education. In the introduction, the NCTM describes the motivation behind the document, stating: "Everyone needs to be able to use mathematics in his or her personal life, in the workplace, and in further study. All students deserve an opportunity to understand the power and beauty of mathematics" (*Principles and Standards*, 2000). Christian mathematics teachers would certainly agree with this statement and should be interested in teaching strategies and tools that can help achieve this goal.

When used properly, technology is one tool that can help educators and students to reach the goals and expectations of *Principles and Standards for School Mathematics*. In fact, the use of technology was set apart as a separate principle in the NCTM report. The technology principle states, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (*Principles and Standards*, 2000). In the summary of this principle, three main points are made. First, technology can allow students to cover more and develop deeper understanding of mathematics by eliminating some of the mundane arithmetic work and spending the additional time probing underlying concepts. Second, technology can benefit a wide range of student abilities by offering opportunities for differentiated instruction. Finally, technology must be used wisely. Teachers must plan on when and how to use it properly, being careful to avoid its misuse (*Principles and Standards*, 2000).

The technology principle should encourage the education community to investigate the effects and best uses of technology. The use of graphing calculators and other static imaging tools has been investigated in depth. Overall, studies seem to show significant improvement in mathematical achievement when these tools are used (Heller, Curtis, Jaffe, & Verboncoeur,

2005; Kulik, 2003). In more recent years, dynamic computer programs such as Geometer's

Sketchpad and Cabri have been introduced to the mathematics community. Numerous studies

have investigated the effects of dynamic computer programs when used in geometry classrooms.

Again, the general consensus is that these programs help achieve geometrical understanding,

although the results tend to be a little more mixed than static imaging tools. (Almeqdadi, 2000;

Hannafin & Scott, 2001). While thoroughly studied in geometry classes, few studies have

examined how computer programs in algebra classrooms affect understanding. Because newer

graphing calculators have dynamic capabilities and dynamic computer programs are becoming

less expensive, the need to study the effects of these tools in algebra classrooms clearly exists.

Problem Statement

Knowledge about the effect of dynamic computer programs on student achievement and

interest in algebra classes is lacking. This study hopes to examine the effectiveness of these

programs in the algebra classroom. Specifically, this study will seek to provide an answer to the

following question: Does the use of Geometer's Sketchpad increase the understanding and

interest of high school algebra students?

Research Questions

To answer the above overarching question, the following research questions will be

addressed in this study.

1. Do the dynamic features of Geometer's Sketchpad in the algebra classroom increase

   student interest more than traditional static image technology based on posttest results?

2. Do the dynamic features of Geometer's Sketchpad in the algebra classroom increase

   student achievement more than traditional static image technology based on posttest

   results?

3. Do the dynamic features of Geometer's Sketchpad in the algebra classroom increase student achievement more than traditional static image technology based on pretest/posttest results?

Definition of Terms

For the purpose of this research study, the following definitions will be used. Unless otherwise noted, the definitions provided are those of the author.

Computer algebra system (CAS) – a program that simplifies algebraic formulas; in addition, many are able to produce static images of functions

Dynamic computer program (DCP) – a computer program that provides dynamic images

Dynamic image – an image that can be manipulated or changed instantly by modifying parameters on the screen or dragging parts of the image.

High-Level Questions – questions that probe conceptual understanding.

Low-Level Questions – questions that replicate questions seen on homework.

Static image – an image that cannot be changed instantly; a user must change the settings in a different menu before the image will change.

<div align="center">

Literature Review

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"The overall aim of Christian schools is to help students become citizens of the Kingdom of God, responsive disciples of Jesus Christ" (Van Brummelen, 1998, p. 26). To meet this goal of Christian education, teachers need to know their students and know ways to reach them. Christian teachers believe that "God created us as image bearers: unique, with special gifts to serve God and our fellow human beings" (Fowler, Van Brummelen, & Van Dyk, 1990, p. 139). Because each student is unique, teachers need to understand that children learn at different rates and stages. "Four interrelated dimensions: guiding, unfolding, structuring, and enabling" is one

process that offers students several ways to understand an idea and allows students time to fully

master the concept (Van Brummelen, 1998, p. 58). Although this is only a small part of a

Christian philosophy of education, it is an integral part of reaching mathematics students because

the understanding of math concepts often develops over time. Christian teachers should desire to

know strategies and tools that can offer students opportunities to ponder the subject.

Christian teachers believe that "learning is meaningful only if the wholeness of life and the

unity of creation become clear to the student" (Steensma & Van Brummelen, 1977, p. 20). In

mathematics, Christian teachers reach this goal in two main ways. Steensma and Van

Brummelen (1997) said, "Mathematics instruction must lead to a responsible *understanding of*

and *participation in* God's created order" (p. 140, emphasis mine). First, teachers want students

to understand a created order. Students should see the connections between strands of

mathematics; that vision allows one to see the beauty and order of mathematics and the Creator

behind it. Second, mathematics must involve participation in created order. One way to do this

is to make math reality-oriented; doing this opens a window to think about stewardship.

Steensma andVan Brummelen (1997) stated that math programs should result in students who

"appreciate how [mathematics] may be used in fulfilling our cultural mandate as Christians" (p.

146). Jongsma summed up both mathematics objectives: "The whole point of studying

mathematics is to develop new and deeper insights into the creation we live in" (Jongsma, 2006,

p. 14). Tools that can help make these two connections should be utilized by Christian

mathematics teachers.

The NCTM's *Principles and Standards for School Mathematics* (2000) is one tool available

to Christian mathematics teachers. The principles in this document demand high expectations

for all students, regardless of their mathematical abilities. All students should be challenged and

be able to succeed in a math classroom. In addition, the principles state a desire for students to see connections among different topics rather than viewing math as a series of unrelated ideas and algorithms. Students should understand concepts and take control of their own learning. The principles encourage teachers to stay up to date with pedagogical tools and ideas, especially technology. Math teachers should see the value in these principles and desire strategies and tools that allow them to implement the principles into their classrooms (*Principles and Standards*, 2000).

Technology in the math classroom is an important way to address the Christian view of the learner, the objectives of Christian mathematics education, and the overall view of the NCTM's principles. The technology principle states, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (*Principles and Standards*, 2000). Although technology can be very useful, it must be implemented wisely. "Before putting monetary resources from tight Christian school budgets into the curriculum integration of computers, we must consider if students will benefit educationally as a result. This means examining the educational research on computing technology and learning" (Van Gorp, 1998, p. 2). In order to be effective, technology must be implemented in the correct way. When contemplating technology use in the classroom, it is important to remember that "it is how the computer is used that is important" (Van Gorp, 1998, p. 3). Christian educators must always remember that students "learn from thinking with God-given minds. The development and nurturing of this thinking, in turn, can be supported by computers" (Van Gorp, 1998, p. 3).

Research shows that computer technology can support student thinking. Graphing calculators and other computer algebra systems (CAS) that provide static images have been

investigated in depth. Kulik (2003) performed a meta-analysis on twelve recent studies that examined conceptual understanding. The results of this study showed an overwhelming positive academic effect on the use of graphing calculators and CAS in the math classroom. Heller, Curtis, Jaffe, and Verboncoeur (2005) investigated the use of graphing calculators and student achievement in Algebra 1 classes and also found that the use of technology led to higher achievement scores. Mayes (2001) found that the CAS's ability to provide multiple representations of functions allowed student to examine problems numerically, graphically, and analytically. This led to deeper probing and understanding. Technology that produces static images has a profoundly positive effect on mathematical understanding.

The introduction of dynamic computer programs (DCP) has introduced a new area of investigations. These types of programs have been in existence for a much shorter time. Most of the studies examine the effects of DCP in geometry classrooms. Hannafin and Scott (2001) found that low achievers in school mathematics performed better when using DCP than those that did not. Almeqdadi (2000) found that the gains made by a group using a DCP were significantly greater than a group that did not use a DCP. The use of DCP in geometry classrooms also seems to be very beneficial.

A natural progression in the use of this technology is to use DCP in algebra classrooms. Fewer studies have examined this topic. Cavanaugh, Gillan, Bosnick, Hess, and Scott (2008) simply found that learning did occur when DCP was used in algebra classrooms, but they could not show that it was better than a more traditional teaching method. Daher (2009) found that students seemed to enjoy using DCP in algebra settings. This area needs to be examined in more depth, especially since graphing calculators like the TI-Nspire series are starting to feature dynamic options and will become much more popular in the future.

Methodology

Participants

Four advanced algebra classes at a midsized suburban Christian high school formed the center of this study. The students in these classes were sophomores and juniors. Class 1 contained 24 students, Class 2 contained 25 students, Class 3 contained 19 students, and Class 4 contained 25 students. All students were familiar with TI-83 or TI-84 graphing calculators, which produce static images. They were not very familiar with Geometer's Sketchpad.

Methods

The classes spent four days learning about linear systems. Day 1 focused on graphing systems of linear equations, Day 2 focused on graphing systems of inequalities, and Day 3 focused on linear programming (this process asks one to graph a system of constraints or inequalities, and then find the minimum or maximum based on the problem's situation and an objective function). Day 4 was dedicated to going over homework assignments and reviewing. Day 5 involved the posttest.

A quasi-experimental methodology was used, and the study followed a Solomon randomized four-group design, as shown in the figure that follows. Classes 1 and 2 used the dynamic features of Geometer's Sketchpad. Classes 3 and 4 utilized the more traditional graphing calculator and the static images that it creates. A pretest was given to Classes 1 and 4 a week prior to this unit of study.

Figure 1: Solomon Randomized Four-Group Design

C_1	$\rightarrow O_1 \rightarrow$	$X_1 \rightarrow$	O_3
C_2	$\rightarrow$	$X_2 \rightarrow$	O_4
C_3	$\rightarrow$		O_5
C_4	$\rightarrow O_2 \rightarrow$		O_6

Key

C = Class
O = Observation or Test
X = Instrument
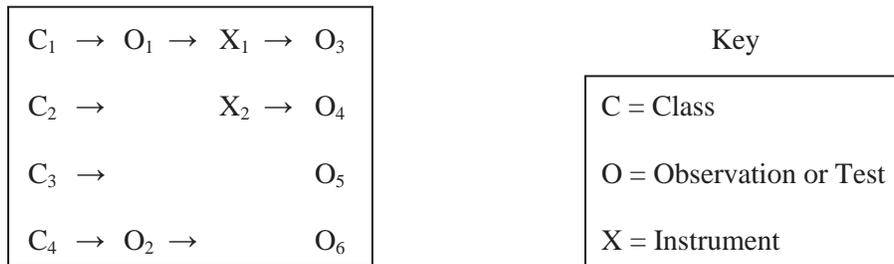
Materials

All four classes used the same general lesson plans. A guided exploration was used for the first three days of the unit. The only difference was the graphing tools that each class used. Notes, activities, and homework assignments were the same for all four classes, although certain handouts had slightly different directions about how to use the graphing tools (i.e. How to plot a function, how to change the viewing window, etc.)

All four classes were given the same posttest at the conclusion of the study. The posttest included two parts: low-level questions and high-level questions. The low-level questions were about basic facts of linear systems that had been covered in the lessons; 6 points could be earned. The high-level questions focused on deeper conceptual understanding; 11 points could be earned. A total of 17 point could be earned on the test. The pretest contained the exact same wording and point totals as the posttest, but the numbers in the problems were changed.

Two precalculus classes took the posttest about a month prior to this study. These students were juniors and seniors and had taken the Advanced Algebra course the year before. Based on their feedback, minor changes were made to the wording of a few questions. No other changes were deemed necessary for the posttest.

A survey on student attitudes about technology in the math classroom was given with the posttest. This survey was written and tested by Reznichenko (2007a); it was deemed an effective measurement in her study. Phrases about technology in the math classroom were written in both positive and negative lights. Students were asked to rank these phrases on a scale of 1 to 5, with 1 being strongly agree and 5 being strongly disagree. The positive phrase results were reverse transposed (1 became a 5, 2 became a 4, etc.). The final result of a 5 meant students viewed technology as beneficial while a final result of a 1 meant that students did not enjoy the use of technology.

Results

Data Analysis and Findings

First, individual class scores were examined by looking at overall scores, low-level scores, high-level scores, and attitude scores. Means and standard deviations of each class were compiled. No significant tests were done with this data, but it is worth noting that all four classes have similar statistics.

Table 1: Means of Individual Classes

	Class 1	Class 2	Class 3	Class 4
Overall	7.958333	7.16	7.684211	8.125
Low-Level	2.75	2.24	2.789474	3.125
High-Level	5.208333	4.92	4.894737	5
Interest Level	3.530797	3.535652	3.883295	3.610507

Table 2: Standard Deviations of Individual Classes

	Class 1	Class 2	Class 3	Class 4
Overall	3.236734	2.838427	4.014593	3.627342
Low-Level	1.259745	1.479865	1.812884	1.541104
High-Level	2.340181	1.823915	2.64354	2.413729
Interest Level	0.670625	0.653467	0.476513	0.848757

Second, the posttest scores from the two Sketchpad classes were combined into a large group; the same thing was done with the graphing calculator classes. The mean and standard deviation of each category was found. The results are in tables 3 and 4.

Table 3: Means of Combined Classes

	Sketchpad Classes	Graphing Calculator Classes
Overall	7.55102	7.930233
Low-Level	2.489796	2.976744
High-Level	5.061224	4.953488
Interest Level	3.533274	3.731041

Table 4: Standard Deviations of Combined Classes

	Sketchpad Classes	Graphing Calculator Classes
Overall	3.034944	3.763201
Low-Level	1.386279	1.654552
High-Level	2.075734	2.487622
Interest Level	0.654992	0.714563

An independent, 1-tailed t-test was used with an alpha level of $p < .05$. In this case, the critical value was 1.645. When the test value is above this critical value, the chance of this happening at random is less than 5%. For test values greater than 1.645, the null hypothesis is rejected and it is accepted that the instrument had an effect on results. In this case, no significant results were shown.

Table 5: Critical Values of Combined Classes

	Critical Values
Overall	-0.87464
Low-Level	-2.45884
High-Level	0.363319
Interest Level	-2.11357

Third, difference scores were examined by looking at overall scores, low-level scores, high-level scores, and interest level scores. In this case, only results from the classes that took

pretests were examined. The pretest score was subtracted from the posttest score. Then the

mean and standard deviation of each category was found. The results are in tables 6 and 7.

Table 6: Means of Difference Scores

	Sketchpad Classes	Graphing Calculator Classes
Overall	5.333333	3.958333
Low-Level	2.458333	2.208333
High-Level	2.875	1.75

Table 7: Standard Deviations of Difference Scores

	Sketchpad Classes	Graphing Calculator Classes
Overall	3.546789	5.254225
Low-Level	1.178767	2.105462
High-Level	2.626164	3.824975

An independent, 1-tailed t-test was used with an alpha level of $p < .05$. In this case, the

critical value was 1.711. When the test value is above this critical value, the chance of this

happening at random is less than 5%. For test values greater than 1.711, the null hypothesis is

rejected and it is accepted that the instrument had an effect on results. In this case, overall and

high-level difference scores are significant.

Table 8: Critical Values of Difference Scores

	Critical Values
Overall	1.89921
Low-Level	1.039005
High-Level	2.098632

Discussion

Summary

Technology is changing the way that mathematics is studied in schools. Graphing calculators have been used for the past two decades and many studies exist that prove their effectiveness. In more recent years, dynamic computer programs have been used in geometry classes. Again, studies have shown that these programs can enhance student learning. These dynamic programs are now able to produce graphs of functions that can be used in algebra classrooms. Few studies exist that explore this use of technology in the math classroom.

This study examined if the use of Geometer's Sketchpad increased the understanding and interest of high school algebra students? Specifically, it examined the following three research questions.

1. Do the dynamic features of Geometer's Sketchpad in the algebra classroom increase student interest more than traditional static image technology based on posttest results?

2. Do the dynamic features of Geometer's Sketchpad in the algebra classroom increase student achievement more than traditional static image technology based on posttest results?

3. Do the dynamic features of Geometer's Sketchpad in the algebra classroom increase student achievement more than traditional static image technology based on pretest/posttest results?

Conclusions

Based on the significant tests of class means, the first two research questions are not supported by this study. Classes that used graphing calculators tended to have a more positive

view of technology. They also did better on the posttest, with the exception of high-level questions.

The third research question is partially supported by this study. Based on the significant tests of difference score class means, the Sketchpad classes produced better difference scores on overall questions and high-level questions.

Implications

The implications of this study are important for mathematics educators. First, it should be noted that Sketchpad classes earned posttest averages that were very close to the graphing calculator classes. Second, the higher difference score averages show that Sketchpad classes gained more while working through this short unit. Mathematics educators can safely assume that there is a sharp learning curve when using this new technology, but students are still able to produce similar results in the end. In the future, mathematics educators are encouraged to carry out similar studies once this technology is more familiar to students.

Limitations

It is quite possible that the newness of Geometer's Sketchpad caused student frustration and affected their attitudes towards technology in the mathematics classroom. While these students had used Sketchpad in the past, these experiences were just one day projects that were few and far between. Students had been using graphing calculators on a daily basis for the entire year, and the abrupt switch in types of technology may have been frustrating for students.

Another limitation for this study may have been the length of time. Perhaps four days is simply not enough time to learn a new type of technology and adequately judge the effectiveness of it. Future studies should either ensure that students are equally familiar with different types of technology or allow for enough time to get past the initial learning curve of the new technology.

Finally, class compositions probably played a small role in the results of this study. Class compositions were beyond the control of the researcher. According to the teacher of the advanced algebra courses, Classes 3 and 4 routinely performed better during the semester than Classes 1 and 2. Classes 1 and 2 were picked to be Sketchpad classes due to the availability of the school's computer labs.

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Appendix A

Parent Letter

Dear Algebra 2 Parents,

The purpose of this note is to inform you of a study being conducted next week in all Algebra 2 classes. Mr. Dekker and Mr. Timmer are investigating different types of graphing technology. During a week-long unit, two classes will utilize graphing calculators and two classes will use a graphing program on a computer. Careful planning has been taken to ensure that all classes cover the main goals of this unit. Students will receive the same activities, assignments, and quizzes. The study will investigate how different types of graphing technologies enable students to grasp higher level concepts. Overall class averages will be used by Mr. Dekker in his Master's thesis. All individual information will be kept confidential; individual results will not be written about in this study. If you have any concerns about this study, please email Mr. Dekker at ddekker@calvinchr.org.

Sincerely,

Mr. Dekker
Mr. Timmer

Appendix B

Using Graphing Calculators to Solve Systems of Equations

Based on "Exploring Algebra 2 with The Geometer's Sketchpad" by Key Curriculum Press

The graphical solution to a system of two equations is the intersection of the graphs of the equations. In real life, you can often solve a system of equations to make a calculated choice between two options. In this activity, you'll solve a system of equations to make some decisions for a truck rental company.

The end of the month is a popular time to move, so many people rent moving trucks. Each truck rental company has its own formula to determine the price of renting a truck. The rental rates depend on two factors: the *Drive-off Fee* and the *Price per Mile Driven*. Use the table below to compare the rates of two different companies.

Rate Comparison for 14-foot Moving Trucks

Company	Drive-off Fee	Price per Mile Driven
Cercano Rental	$12.00	$2.75
We-Haul-It	$42.50	$0.40

With the rate information, the renter can make an informed decision. The price of renting a 14-foot truck will depend on the number of miles driven.

Q1. **What is the price formula for Cercano Rentals?**

Q2. **What is the price formula for We-Haul-It?**

Which company is cheaper depends on how many miles you drive. To compare the prices, you can graph each equation and find the intersection of the graphs.

A.   Select WINDOW and set Xmin and Ymin to 0. (Does this make sense?) Set the Xmax and Ymax values so that you get the window that seems right to you.

B.   Now, select Y= and plot the function from Q1 in Y1.

C.   Plot the function from Q2 in Y2.

D.   Adjust your window if it doesn't seem right to you.

Q3. **Estimate the x and y values at the intersection point. Don't forget labels!**

E.   Select TRACE and use the arrow keys to find the intersection.

Q4. **Was your estimation in Q3 close to this new intersection estimation? Modify your estimate if necessary.**

F.   Now we want to find exact coordinates of the intersection. Select 2nd→TRACE and select intersect. Hit enter twice to select the two lines, then use the arrows to estimate the intersection, and hit enter a third time. If done correctly, the intersection coordinates should appear.

Q5. **Write down the actual intersection with labels. What is the significance of this point?**

Q6. **Why do you think the point of intersection is often called the *break-even point*?**

Q7. **If the renter of the moving truck will drive 10 miles, from which company should he rent?**

Q8. **In Spanish, cercano means "nearby." Do you think that the name Cercano Rentals is appropriate for the way people use its trucks?**

The owners of Cercano Rentals would like the company to be more competitive in moves of longer distances. The company investigates how it can change its rates to attract more long-distance movers. For marketing reasons, the company believes it is important to maintain its low *Drive-off Fee* of $12.00. Therefore, it decides to explore different scenarios by adjusting its *Price per Mile Driven*.

G.   As you work through this section, an error message might mean you have to expand your Xmax and Ymax values.

H.   Select Y= and change the 2.75 value in Y1 to 2.50.

I.   Find the intersection of the two equations.

I.   Repeat steps H and I for values of 2.25 and 2.00.

J.   Fill in the chart provided and summarize what increasing the slope does for the intersection. Make sure your summary includes labels for the three variables involved.

Slope in Y1	2.75	2.50	2.25	2.00
X-value of intersection				
Y-value of intersection				

Summary:

Q9. **Approximately what Price per Mile Driven should Cercano Rental charge so that it is more competitive for moves of…**

Less than 50 miles?   **Less than 100 miles?**

Q10. **What price would Cercano Rental charge so that they are always cheaper than We-Haul-It?**

Appendix C

Using Graphing Calculators to Solve Systems of Inequalities

Based on "Exploring Algebra 2 with the Geometer's Sketchpad" by Key Curriculum Press

The Problem

Sally volunteers to bring origami swans and giraffes to sell at a charity crafts fair. It takes her three minutes to make a swan and six minutes to make a giraffe. She plans to sell the swans for $4 each and the giraffes for $6 each. She has only 16 pieces of origami paper and can't spend more than one hour folding. How many of each animal can Sally make that will still meet all of these requirements?

Assign Variables and Write the Equation.

There's a lot of information to deal with here. The actual question is "How many of each type of animal can Sally make?" so start by assigning variables:

Let x = the number of swans Sally makes.

Let y = the number of giraffes Sally makes

Q1.   Write an equation for profit in terms of x and y.

A.   Type the equation into your home screen.

B.   Select WINDOW and set your window so that it is a square with all four quadrants shown evenly. Then select GRAPH.

C.   Select TRACE, use the arrows to move the cursor around, go back to the home screen, and select ENTER. Your calculator will plug the X and Y values from the TRACE into the equation and give you the profit. Repeat this several times using different TRACE points.

Q2.   What kind of ordered pairs make sense for this situation?

Q3.   In what quadrant is this equation useful? How would you describe this quadrant in terms of x and y?

The First Constraint Inequality

Other constraints or limitations also exist. The first constraint is that Sally has only 16 pieces of origami paper. We'll call this the paper constraint and express it with an inequality.

Q4.   Write an inequality for the paper in terms of x and y.

D.   We want to graph this inequality. First, solve for y and write the equivalent inequality here.

E.   Graph the equation related to this inequality. Select Y= and type the equation into Y1. View the graph.

F.   Select TRACE and move the point on the line. Select points above and below the line and manually plug them into the paper inequality.

Q5.   Where are all constraints met?

G.   To graph the inequality, go back to Y=, move the cursor to the far left of Y1, and hit ENTER several times until you pick the right inequality.

H.   Go to the graph again and select points within this region. Manually plug them into the profit equation.

Q6.   Within this region, where is the profit greatest?

I.   Select Y= and move the cursor over the = symbol in Y1. Hit ENTER so that the = symbol is no longer highlighted. This will hide the inequality for the next part of the activity.

The Second Constraint Inequality

Q7.   Write an inequality for time in terms of x and y. Remember to use the same units.

J.   Follow the same instructions in steps D-F, but use Y2 instead of Y1.

Q8.   Where are all constraints met?

K.   Follow the same instructions in steps G-H.

Q9.   Within this region, where is the profit greatest?

Find the Feasible Region

L.   Go to Y= and highlight the = symbol in Y1. This will make the first inequality appear again.

Q10. What new pattern appears? What does it represent?

Q11. Why don't the two remaining "single-lined" sections work?

M.   Write down exactly what you see in Y= and WINDOW so you can use it again tomorrow.

Appendix D

Linear Programming with Graphing Calculators

In business, it's often difficult to figure out how to maximize profit because there are so many factors to consider: labor costs, time constraints, production capacity, etc. Linear programming is a branch of algebra that helps deal with complicated situations such as this. Everything we did yesterday is part of the linear programming process.

A.   Review the Activities from yesterday. Any questions?

The purpose of linear programming is to find the best possible solution to a problem. How many of each animal should Sally make to maximize the charity's profit?

Find the Maximum Value

The checked region you found in Q10 represents where both constraints are satisfied. It is called the *feasible region*. Your goal now is to find the one point within this region that maximizes profit. You can test various points and try to find where the highest profit occurs, but there is an easier way. Linear programmers can prove that *the maximum value always occurs at a corner point of the feasible region*. But which corner is it?

B.   We want to test the four corners of this green region. Use your brain or your calculator (or a combination of the two) to find the coordinates for the origin, y-intercept, x-intercept, and intersection of the lines. Test all four points to make sure they meet the constraints.

Q1.   Where is the profit the greatest? How many of each animal should Sally make?

C.   Let's examine why this point is the maximum. Take the profit equation from Q1 and solve for y. Graph this equation in Y3, leaving x and y as variable but inserting the 20 into the Profit part of the equation.

Q2.   What do the ordered pairs on this line in the checked section represent?

D.   Go back to Y= and CAREFULLY replace 20 with 25, then view the graph. Continue this process several times, each time increasing the profit by 5.

Q3.   What part of the checked section does the Profit equation seem to zero in on as the "Profit" parameter increases?

Review of Linear Programming

(Answers for this activity or in parentheses)

1. Decide what you are maximizing or minimizing. (Profit)

2. Assign variables for the items that determine the maximum or minimum.

    (Swans [x] and Giraffes [y])

2. Find Maximum or Minimum Equation using the variables. (P = 4x + 6y)

3. Find constraint inequalities and graph them. (x + y $\leq$ 16   and   3x + 6y $\leq$ 60)

4. Find the Feasible Region. (The checked area)

5. Test the corner points of the feasible region to find the minimum or maximum.

    (12 swans and 4 giraffes)

Appendix E

Using Sketchpad to Solve Systems of Equations

Based on "Exploring Algebra 2 with The Geometer's Sketchpad" by Key Curriculum Press

The graphical solution to a system of two equations is the intersection of the graphs of the equations. In real life, you can often solve a system of equations to make a calculated choice between two options. In this activity, you'll solve a system of equations to make some decisions for a truck rental company

The end of the month is a popular time to move, so many people rent moving trucks. Each truck rental company has its own formula to determine the price of renting a truck. The rental rates depend on two factors: the *Drive-off Fee* and the *Price per Mile Driven*. Use the table below to compare the rates of two different companies.

Rate Comparison for 14-foot Moving Trucks

Company	Drive-off Fee	Price per Mile Driven
Cercano Rental	$12.00	$2.75
We-Haul-It	$42.50	$0.40

With the rate information, the renter can make an informed decision. The price of renting a 14-foot truck will depend on the number of miles driven.

Q1.   What is the price formula for Cercano Rentals?

Q2.   What is the price formula for We-Haul-It?

Which company is cheaper depends on how many miles you drive. To compare the prices, you can graph each equation and find the intersection of the graphs.

A.   Select GRAPH→Define Coordinate System. Decide what your x-axis and y-axis represent. Drag the origin to the bottom left corner of your screen. (Does this make sense?) Now grab one of the numbers on the x-axis and move it left or right until you get the window that seems right to you.

B.   Now, plot your first function. Select GRAPH→Plot New Function. Enter the function from Q1 and hit OK.

C.   Plot your second function. Follow the directions in Step B, but this time use the function in Q2.

D.   Adjust your window if it doesn't seem right to you.

Q3.   Estimate the x and y values at the intersection point. Don't forget labels!

E.   Select the point tool in the top left corner. Construct a point in the first quadrant. With only the point selected, select MEASURE→Coordinates. Move the point until it is close to the intersection and note the coordinates.

Q4.   Was your estimation in Q3 close to this new intersection estimation? Modify your estimate if necessary.

F. Now we want to find exact coordinates of the intersection. Open up Intersections.gsp from Mr. Dekker's Pick Up folder. (Note: this file will always be here for your use.) You must have this file open to use a special tool that finds the intersections of equations. Go back to your graph by choosing WINDOW →Your File. To pick the custom tool, select the bottom button from the top left toolbar. Hold the button down, select INTERSECTIONS→Find Intersection. Select one function (f(x) = …, not the line), then the other. Now place a point on one of the lines on the far left side of the screen, then on the same line on the far right side of the intersection. If done correctly, exact coordinates of the intersection should appear.

Q5. **Write down the actual intersection with labels. What is the significance of this point?**

Q6. **Why do you think the point of intersection is often called the *break-even point*?**

Q7. **If the renter of the moving truck will drive 10 miles, from which company should he rent?**

Q8. **In Spanish, cercano means "nearby." Do you think that the name Cercano Rentals is appropriate for the way people use its trucks?**

The owners of Cercano Rentals would like the company to be more competitive in moves of longer distances. The company investigates how it can change its rates to attract more long-distance movers. For marketing reasons, the company believes it is important to maintain its low *Drive-off Fee* of $12.00. Therefore, it decides to explore different scenarios by adjusting its *Price per Mile Driven*.

G. First we want to create a parameter that represents Price per Mile Driven. Select GRAPH→New Parameter and name it Per Mile and set the value at 2.75. Then hit OK.

H. Right click on the parameter and select Properties. Select Parameter and change Keyboard Adjustments to .05.

I. Now right click on the Cercano function and select Edit Function. Delete 2.75. Before you move anything else, click on the Per Mile parameter. It should now appear in the function. Select OK

J. Click the Per Mile parameter. Then hit the + key several times and note what happens. Then hit the – key and note what happens.

K. Fill in the chart provided and summarize what increasing the slope does for the intersection. Make sure your summary includes labels for the three variables involved.

Slope in Y1	2.75	2.50	2.25	2.00
X-value of intersection				
Y-value of intersection				

Summary:

Q9. **Approximately what Price per Mile Driven should Cercano Rental charge so that it is more competitive for moves of…**
Less than 50 miles?   Less than 100 miles?

Q10. **What price would Cercano Rental charge so that they are always cheaper than We-Haul-It?**

Appendix F

Using Sketchpad to Solve Systems of Inequalities

Based on "Exploring Algebra 2 with the Geometer's Sketchpad" by Key Curriculum Press

The Problem

Sally volunteers to bring origami swans and giraffes to sell at a charity crafts fair. It takes her three minutes to make a swan and six minutes to make a giraffe. She plans to sell the swans for $4 each and the giraffes for $6 each. She has only 16 pieces of origami paper and can't spend more than one hour folding. How many of each animal can Sally make that will still meet all of these requirements?

Assign Variables and Write the Equation.

There's a lot of information to deal with here. The actual question is "How many of each type of animal can Sally make?" so start by assigning variables:

Let x = the number of swans Sally makes.   Let y = the number of giraffes Sally makes

Q1.   Write an equation for profit in terms of x and y.

A.   Create a coordinate plane and place a point in it. Highlight the point.

B.   Select MEASURE→Abscissa(x). Right-click on the measurement, choose Label Measurement, and label it Swans.

C.   Highlight the point again. Select MEASURE→Ordinate(y), then label it Giraffes.

D.   Select MEASURE→Calculate and type in your profit equation from Q1. Click the Swan measurement instead of x and the Giraffe measurement instead of y.

E.   Create a text box in front of the calculation that says "Profit (dollars) ="

Q2.   Move the point around and note the measurements. What kind of ordered pairs make sense for this situation?

Q3.   In what quadrant is this equation useful? How would you describe this quadrant in terms of x and y?

The First Constraint Inequality

Other constraints or limitations also exist. The first constraint is that Sally has only 16 pieces of origami paper. We'll call this the paper constraint and express it with an inequality.

Q4.   Write an inequality for the paper in terms of x and y.

F.   We want to graph this inequality. First, solve for y and write the equivalent inequality here.

G.   Graph the equation related to this inequality. Select GRAPH→Plot New Function and type in the related equation.

H.   Repeat steps D and E, this time using an equation that shows how much paper is being used. Use the label "Paper (pieces) ="

Q5.   Move the point around. Where are all constraints met?

I.   To graph the inequality, we need to find 3 points (the origin, the y-intercept, and the x-intercept). The origin is already found for you.

J.   To find the y-intercept, look at the inequality in F. To plot the point, select GRAPH→Plot Points and type in the point.

K.   To find the x-intercept, first graph the line y=0. Then open up Intersection.gsp from Mr. Dekker's Pick Up folder and find the intersection of the two lines.

L.   Select the three points in order. Then select CONSTRUCT→Triangle Interior. Make the color yellow.

Q6.   Move the point around. Within this region, where is the profit greatest?

M.   Make sure the triangle, the paper equation, the label, and the line are the only things highlighted. Select EDIT→Action Buttons→Hide/Show. Click the button to hide the inequality for the next part of the activity.

<div align="center">

The Second Constraint Inequality

</div>

Q7.   Write an inequality for time in terms of x and y. Remember to use the same units.

N.   Follow the same instructions in steps F-H, but use the label "Time (minutes) =" instead.

Q8.   Move the point around. Where are all constraints met?

O.   Follow the same instructions in steps I-L, but make the triangle light blue.

Q9.   Move the point around. Within this region, where is the profit greatest?

<div align="center">

Find the Feasible Region

</div>

P.   Click the Hide/Show button to show the first inequality again.

Q10.   What new color appears? What does it represent?

Q11.   Why don't the remaining yellow and blue sections work?

Q.   Save the file in your student account so that you can use it again tomorrow.

Appendix G

Linear Programming with Sketchpad

In business, it's often difficult to figure out how to maximize profit because there are so many factors to consider: labor costs, time constraints, production capacity, etc. Linear programming is a branch of algebra that helps deal with complicated situations such as this. Everything we did yesterday is part of the linear programming process.

A.   Review the Activities from yesterday. Any questions?

The purpose of linear programming is to find the best possible solution to a problem. How many of each animal should Sally make to maximize the charity's profit?

Find the Maximum Value

The green region you found in Q10 represents where both constraints are satisfied. It is called the *feasible region*. Your goal now is to find the one point within this region that maximizes profit. You can test various points and try to find where the highest profit occurs, but there is an easier way. Linear programmers can prove that *the maximum value always occurs at a corner point of the feasible region*. But which corner is it?

B.   We want to test the four corners of this green region. Use your brain or the tools of Sketchpad (or a combination of the two) to find the coordinates for the origin, y-intercept, x-intercept, and intersection of the feasible region. Test all four points to make sure they meet the constraints.

Q1.   Where is the profit the greatest? How many of each animal should Sally make?

C.   Let's examine why this point is the maximum. Select GRAPH→New Parameter. Label it as Profit and set the initial value for 20.

D.   Take the profit equation from Q1 and solve for y. Graph this equation, leaving x and y as variable but clicking on the Profit parameter for that part of the equation.

Q2.   What do the ordered pairs on this line in the green section represent?

E.   Click the Profit parameter and hit the "+" key several times.

Q3.   What part of the green section does the Profit equation seem to zero in on as the "Profit" parameter increases?

Review of Linear Programming

(Answers for this activity are in parentheses)

1. Decide what you are maximizing or minimizing. (Profit)

2. Assign variables for the items that determine the maximum or minimum.
   (Swans [x] and Giraffes [y])

2. Find Maximum or Minimum Equation using the variables. (P = 4x + 6y)

3. Find constraint inequalities and graph them. (x + y $\leq$ 16   and   3x + 6y $\leq$ 60)

4. Find the Feasible Region. (The green area)

5. Test the corner points of the feasible region to find the minimum or maximum.
   (12 swans and 4 giraffes)

Appendix H

Systems of Equations Homework

For Questions 1 and 2, solve each system by graphing using technology. Sketch and label a graph below.

1. $\begin{cases} y = x - 2 \\ y = -2x + 7 \end{cases}$

2. $\begin{cases} -5x + y = -9 \\ x + 3y = 21 \end{cases}$

3. To pay your monthly bills, you can either open a checking account or use an online banking service. A local bank charges $3 per month and $.40 per check, while an online service charges a flat fee of $9 per month.

   a. Write a system of linear equations to model the cost c of each service for b bills that you need to pay monthly and graph using technology. Sketch and label a graph below.

   b. Find the point of intersection of the two linear models. What does this answer represent?

   c. If you pay about 12 bills per month, which service should you choose? Explain.

   d. The bank discovered that most people wrote a lot of checks per month. They wanted to be more competitive to clients who had a lot of bills to pay. They want to keep their $3 monthly charge. What should their per check price be if they want the new equilibrium point to be at 25 checks?

4. Suppose you are going on vacation and leaving your dog in a kennel. The Bowowery charges $25 per day, which includes a one-time grooming treatment. The Poochpad charges $20 per day and a one-time fee of $30 for grooming. Your vacation is a week long. Graph and solve a system of equations using technology to that models this situation. Which company did you choose? Explain. Please sketch all graphs and show all work.

5. You and your business partner are mailing advertising flyers to your customers. Both of you have started addressing them separately, but decide to finish them together. You address 6 flyers each minute and have already done 80. Your partner addresses 4 flyers each minute and has already done 100. Graph and solve a system of equations using technology to find when the two of you will have addressed an equal number of flyers. Please sketch all graphs and show all work.

Appendix I

Systems of Inequalities Homework

For Questions 3-4, decide if (-3, 3) is a solution of the system without using technology. Then graph the system and the point using technology to verify your answer. Sketch a graph for each question.

1. $\begin{cases} y \geq x + 2 \\ 3y < -6x + 6 \end{cases}$

2. $\begin{cases} y - 2x \leq 1 \\ y < -2x - 2 \end{cases}$

3. $\begin{cases} y \leq 2x + 2 \\ y < -x + 1 \end{cases}$

4. $\begin{cases} x + 2y \leq 10 \\ x + y \leq 3 \end{cases}$

5. A psychologist needs at least 40 subjects (adults and children) for her experiment. She cannot use more than 30 children. Write a system of inequalities to model the situation and graph it using technology. Sketch and label a graph below.

6. You want to bake at least 6 and at most 11 loaves of bread for a bake sale. You want at least twice as many loaves of banana bread as nut bread. Write a system of inequalities to model the situation and graph it using technology. Sketch and label a graph below.

Appendix J

Linear Programming Homework

1. Find the values of x and y that maximize or minimize the objective function for the graph.
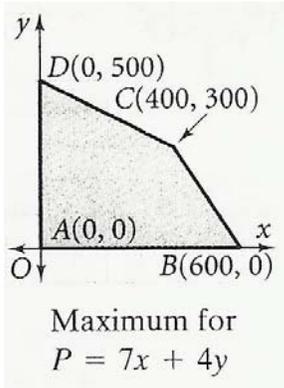
For Questions 2 and 3, graph each system of constraints using technology. Then find the values of x and y that maximize or minimize the objective function.

2.
$$\begin{cases} x + y \geq 8 \\ y \geq 5 \\ x \geq 0 \end{cases}$$
Minimum for P = 3x + 2y

3.
$$\begin{cases} x + y \leq 8 \\ 2x + y \leq 10 \\ x \geq 0, y \geq 0 \end{cases}$$
Maximum for N = 100x + 40y

For Questions 4-7, please write and label all important equations and inequalities. All graphing should be done using technology. Sketch what your coordinate plane looks like and clearly show and label your answers.

4. Baking a tray of corn muffins takes 4 cups of milk and 3 cups of wheat flour. A tray of bran muffins takes 2 cups of milk and 3 cups of wheat flour. A baker has 16 cups of milk and 15 cups of wheat flour. He makes a $3 profit per tray of corn muffins and a $2 profit per tray of bran muffins. How many trays of each type of muffin should the baker make to maximize his profit?

5. Trees in urban areas help keep air fresh by absorbing carbon dioxide. A city has $2100 to spend on planting spruce and maple trees. The land available for planting is 45,000 ft^2. How many of each tree should the city plant to maximize carbon dioxide absorption?

6. Teams chosen from 30 forest rangers and 16 trainees are planting trees. An experienced team consisting of two rangers can plant 500 trees per week. A training team consisting of one ranger and two trainees can plant 200 trees per week. How many of each type of team should be formed to maximize the number of trees planted? How many trainees are used in this solution? How many trees are planted?

7. A biologist is developing two new strains of bacteria. Each sample of Type I bacteria produces four new viable bacteria, and each sample of Type II produces three new viable bacteria. Altogether, at least 240 new viable bacteria must be produced. At least 30, but not more than 60, of the original samples must be Type I. Not more than 70 of the samples can be Type II. A sample of Type I costs $5 and a sample of Type II costs $7. How many samples of Type II bacteria should be used to minimize cost?

Appendix K

Pretest

For Question 1, solve the system by graphing. Sketch and label a graph below.

1. $\begin{cases} 3x + 7y = -6 \\ x + 2y = 11 \end{cases}$

For Questions 2 and 3, classify each system as *independent* (crosses once), *dependent* (the same line), or *inconsistent* (never crosses) without graphing. Explain you answer.

2. $\begin{cases} x + y = 4 \\ -4x + y = 9 \end{cases}$

3. $\begin{cases} 4x - 2y = 6 \\ 6x - 3y = 9 \end{cases}$

4. To develop your digital photos, you can use either EZ Online Digital Photos or a local pharmacy. EZ Online charges $0.20 per digital photo and $2.00 for shipping. The local pharmacy charges $0.30 per digital photo.

    a. Graph and solve a system of equations using technology to determine when it makes more sense to use EZ Online. Explain your answer.

    b. What should the pharmacy charge per picture if it wants to have the same price as EZ Online's price at 100 pictures?

    c. How should EZ Online change their price if they want to keep the $0.20 charge per photo but have the point of equilibrium occur at 15 pictures?

5. Write a second equation in slope-intercept form for the system so that it has no solutions

$\begin{cases} y = 3x + 4 \\ \quad ? \end{cases}$

6. Is it possible for a system of linear equations to have exactly two solutions? Explain.

7. Is it possible for an inconsistent linear system to consist of two lines with the same y-intercept? Explain.

8. Decide if (3, -2) is a solution of the system $\begin{cases} y \geq 3x - 4 \\ 2y + x < 2 \end{cases}$ without using a calculator. Then graph the system and the point to verify your answer.

9. The Merry Bakers are baking pumpkin bread and apple cinnamon bread for this week's specials. A loaf of pumpkin bread requires 3 cups of flour and 1.5 tsp. of baking powder, while a loaf of apple cinnamon bread requires 1.5 cups of flour and 4.5 tsp. of baking powder. The bakers have at most 24 cups of flour and at most 27 teaspoons of baking powder on hand. Write and graph a system of inequalities using technology to model the situation. Quickly sketch and label a graph below.

10. Write and graph a system of 3 inequalities that has no solution.

11. Write a system of inequalities
    to describe the shaded figure.

(0, 3)

(-3, 0)

(3, 0)

(0, -3)
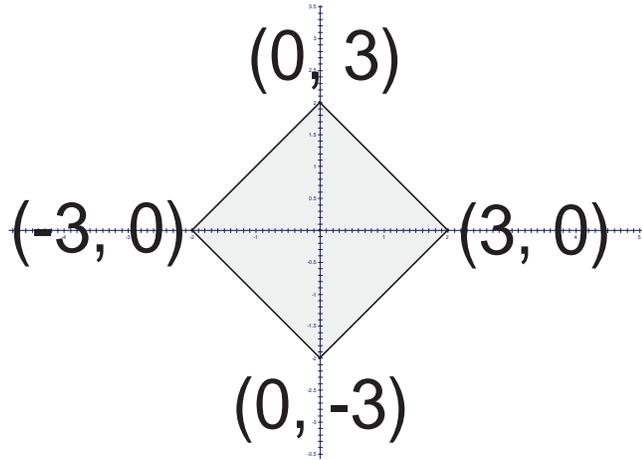
For Question 12, graph the system of constraints using technology. Sketch what your coordinate plane looks like. List all vertices of the feasible region. Then find the values of x and y that maximize or minimize the objective function.

12. $\begin{cases} y \le 2x - 3 \\ y \ge -4 \\ x \le 3 \end{cases}$

    Minimum for P = x − y

For Questions 13 and 14, please write and label all important equations and inequalities. All graphing should be done using technology, but also sketch what your coordinate plane looks like and clearly show and label your answers.

13. The Future Homemakers Club is making canvas tote bags and leather tote bags for a fund-raiser. They will earn $25 or each canvas bag and $32 for each leather bag. They will line both types of tote bags with canvas and use leather for the handles of both. For the canvas bags, they need 6 yards of canvas and 2 yards of leather. For the leather bags, they need 6 yards of leather and 4 yards of canvas. Their advisor purchased 120 yards of leather and 164 yards of canvas. How many of each bag should they make to maximize profit? What is the maximum profit?

14. Farmer Brown can plant up to 70 acres of land with corn and soybeans. He will earn $80 per acre of corn and $90 per acre of soybeans. At least 20 acres of the land must be planted in corn. He will spend up to $13,000 on planting. If it costs $200 to plant an acre of corn and $150 to plant an acre of soybeans, how many acres of each should he plant to maximize profits? What is the maximum profit?

15. Evaluate the profit formula P = 3x + 2y for the shaded area shown. Find three points that yield the maximum profit.
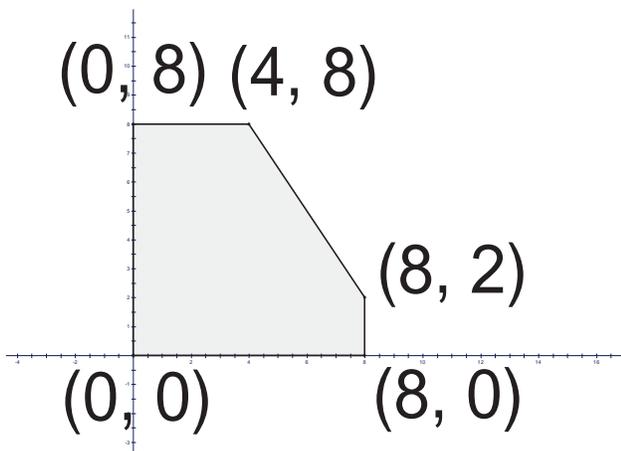
(0, 8) (4, 8)

(8, 2)

(0, 0)   (8, 0)

16. Sally's Sandwich Shop has only one type of sandwich on its menu: ham and cheese. The following graph shows a system of inequalities that represents various constraints to the sandwich's production. State the two inequalities modeled in the graph. Then give a plausible and realistic explanation to what they might mean in this situation. Make sure that your explanation is reasonable and conceivable to the situation

HAM

(0, 63)

(0, 35)

(28, 21)

CHEESE

(0, 0)   (42, 0) (70, 0)
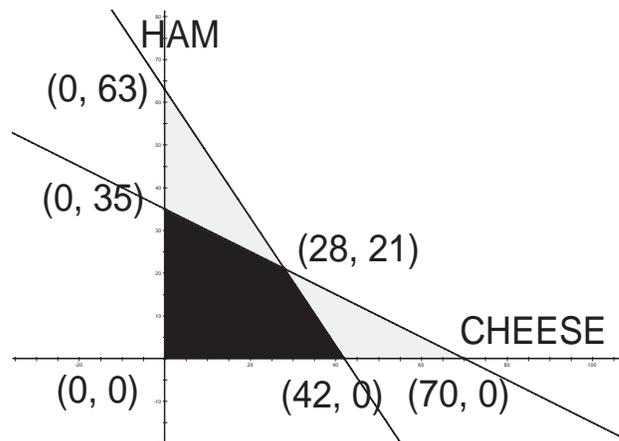
Appendix L

Posttest

For Question 1, solve the system by graphing. Sketch and label a graph below.

1. $\begin{cases} 2x + 3y = 12 \\ 2x - y = 4 \end{cases}$

For Questions 2 and 3, classify each system as *independent* (crosses once), *dependent* (the same line), or *inconsistent* (never crosses) without graphing. Clearly explain how you can know this without graphing.

2. $\begin{cases} 3x + y = 3 \\ 6x + 2y = 6 \end{cases}$

3. $\begin{cases} y - x = 5 \\ 2y - 2x = 8 \end{cases}$

4. To develop your digital photos, you can use either EZ Online Digital Photos or a local pharmacy. EZ Online charges $0.15 per digital photo and $3.00 for shipping. The local pharmacy charges $0.25 per digital photo.

   a. Graph and solve a system of equations using technology to determine when it makes more sense to use EZ Online. Explain your answer.

   b. What should the pharmacy charge per picture if it wants to have the same price as EZ Online's price at 100 pictures?

   c. How should EZ Online change their price if they want to keep the $0.15 charge per photo but have the point of equilibrium occur at 24 pictures?

   5. Write a second equation in slope-intercept form for the system so that it has no solutions
   $\begin{cases} y = 2x - 3 \\ \quad ? \end{cases}$

6. Is it possible for a system of linear equations to have exactly two solutions? Explain.

7. Is it possible for an inconsistent linear system to consist of two lines with the same y-intercept? Explain.

8. Decide if (-2, 5) is a solution of the system $\begin{cases} y \geq 2x + 2 \\ 3y + 2x < 16 \end{cases}$ without using a calculator. Then graph the system and the point to verify your answer. Clearly explain how you can know this without graphing.

9. The Merry Bakers are baking pumpkin bread and apple cinnamon bread for this week's specials. A loaf of pumpkin bread requires 2 cups of flour and 1 tsp. of baking powder, while a loaf of apple cinnamon bread requires 1.5 cups of flour and 2.5 tsp. of baking powder. The bakers have at most 24 cups of flour and at most 26 teaspoons of baking powder on hand. Write and graph a system of inequalities using technology to model the situation. Sketch and label a graph below.

10. Write and graph a system of 3 inequalities that has no solution.

11. Write a system of inequalities to describe the shaded figure.

(0, 2)

(-2, 0)

(2, 0)

(0, -2)
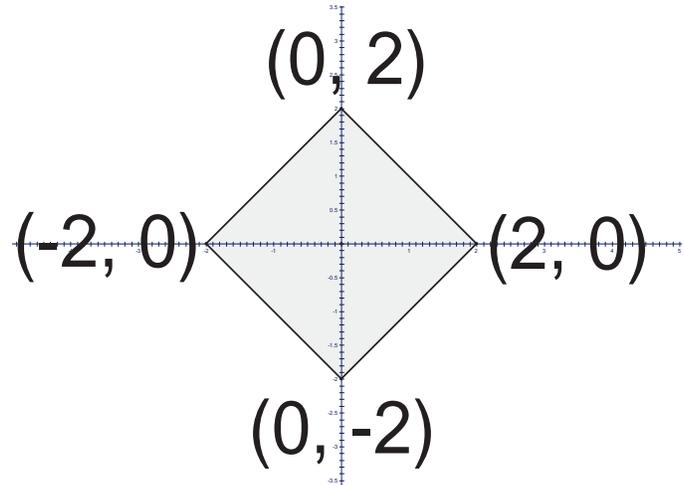
For Question 12, graph the system of constraints using technology. Sketch what your coordinate plane looks like. List all vertices of the feasible region. Then find the values of x and y that maximize or minimize the objective function.

12. $\begin{cases} y \le 3x - 4 \\ y \ge -4 \\ x \le 3 \end{cases}$

Minimum for P = x – y

For Questions 13 and 14, please write and label all important equations and inequalities. All graphing should be done using technology, but also sketch what your coordinate plane looks like and clearly show and label your answers.

13. The Future Homemakers Club is making canvas tote bags and leather tote bags for a fund-raiser. They will earn $20 or each canvas bag and $35 for each leather bag. They will line both types of the tote bags with canvas and use leather for the handles of both. For the canvas bags, they need 4 yards of canvas and 1 yard of leather. For the leather bags, they need 3 yards of leather and 2 yards of canvas. Their advisor purchased 66 yards of leather and 104 yards of canvas. How many of each bag should they make to maximize profit? What is the maximum profit?

14. Farmer Brown can plant up to 50 acres of land with corn and soybeans. He will earn $75 per acre of corn and $85 per acre of soybeans. At least 10 acres of the land must be planted in corn. He will spend up to $12,000 on planting. If it costs $250 to plant an acre of corn and $200 to plant an acre of soybeans, how many acres of each should he plant to maximize profits? What is the maximum profit?

15. Evaluate the profit formula $P = x + 2y$ for the shaded area shown. Find three points that yield the maximum profit.
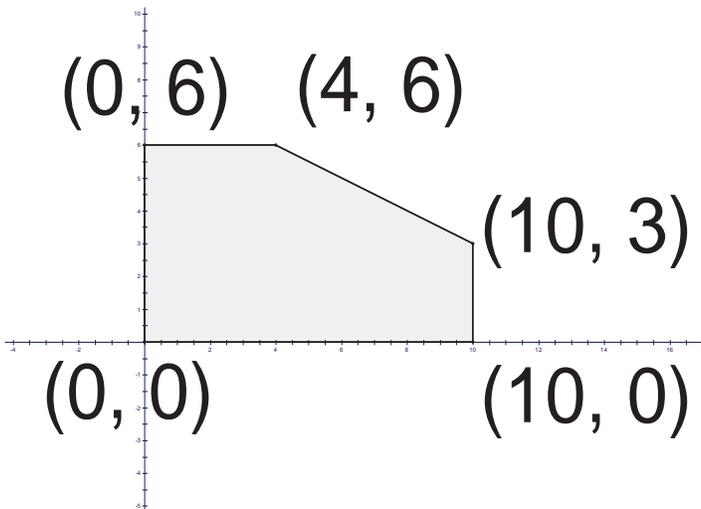
(0, 6)   (4, 6)

(10, 3)

(0, 0)   (10, 0)

16. Sally's Sandwich Shop has only one type of sandwich on its menu: ham and cheese. The following graph shows a system of inequalities that represents various constraints to the sandwich's production. State the two inequalities modeled in the graph. Then give a plausible and realistic explanation to what they might mean in this situation. Make sure that your explanation is reasonable and conceivable to the situation

HAM

(0, 60)

(0, 40)

(30, 20)

CHEESE

(0, 0)   (45, 0) (60, 0)
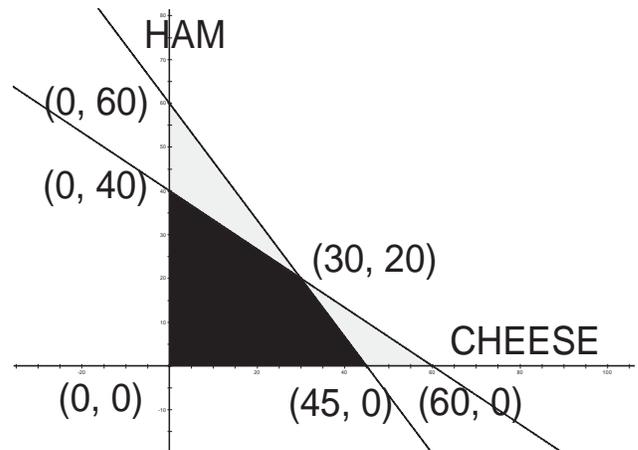
Appendix M

Attitude Survey

Choose one from the following five-point scale:

1 = strongly agree, 2 = agree, 3 = neutral, 4 = disagree, 5 = strongly disagree

1. **1 2 3 4 5**   Computing power makes it easier to explore mathematical ideas.

2. **1 2 3 4 5**   I know technology is important but I don't feel I need to use it to learn mathematics.

3. **1 2 3 4 5**   Technology is a good tool for calculations, but not for my learning of mathematics.

4. **1 2 3 4 5**   I think using technology is too new and strange to make it worthwhile for learning mathematics.

5. **1 2 3 4 5**   I think using technology wastes too much time in the learning of mathematics.

6. **1 2 3 4 5**   I prefer to do all the calculation and graphing myself, without using Sketchpad.

7. **1 2 3 4 5**   Using technology for the calculations makes it easier for me to do more realistic applications.

8. **1 2 3 4 5**   I like the idea of exploring mathematical methods and ideas using technology.

9. **1 2 3 4 5**   I want to get better at using technology to help me with mathematics.

10. **1 2 3 4 5**   The symbols and language of mathematics are bad enough already without the addition of technology.

11. **1 2 3 4 5**   Having technology to do routine work makes me more likely to try different methods and approaches.

12. **1 2 3 4 5**   I don't have any use for technology on a day to day basis.

13. **1 2 3 4 5**   I don't think technology will be useful to me in my future job.

14. **1 2 3 4 5**   Anything that technology can be used for, I can do just as well some other way.

15. **1 2 3 4 5**   I don't see how technology can help me to learn some new skills.

16. **1** **2** **3** **4** **5**   I enjoy using technology in mathematics activities.

17. **1** **2** **3** **4** **5**   Technology is interesting, fascinating, and easy to use.

18. **1** **2** **3** **4** **5**   I enjoy investigating mathematics problems using technology.

19. **1** **2** **3** **4** **5**   Technology is very interesting and challenging to use.

20. **1** **2** **3** **4** **5**   I don't feel comfortable when I learn mathematics using technology.

21. **1** **2** **3** **4** **5**   The thought of using technology in mathematics activities frightens me.

22. **1** **2** **3** **4** **5**   I am worried about using technology because I do not know what to do if something goes wrong.

23. **1** **2** **3** **4** **5**   The use of technology confuses me.

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Education

B.A. Secondary Mathematics Education, Calvin College (2004)

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